Cosmic Evolution as Inertial Motion in the Field Space of GR

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The identification of a cosmic scale function with the volume integral of a spacelike hypersurface defines the cosmic evolution in General Relativity as a collective motion along a geodesic in the field space of the metric components, considered as the coset of the affine group over the Lorentz one. The Friedmann equations are derived out of the homogeneous approximation by the Gibbs averaging exact equations over the relative constant spatial volume.

A direct correspondence between the collective cosmic motion and Special Relativity is established, to solve the problem of time and energy by analogy with the solution of this problem for a relativistic particle by Poincare and Einstein. A geometrical time interval is introduced into quantum theory of the relativistic collective motion by the canonical Levi-Civita – type transformation in agreement with the correspondence principle with quantum field theory. In this context the problem of quantum cosmological creation of visible matter is formulated. We show that latest observational data can testify to the relative measurement standard, and the cosmic evolution as an inertial motion along geodesic in the field space.

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I. INTRODUCTION

The discovery of evolution of the Universe in the form of the Hubble law is considered as one of the greatest achievements of modern physics [1, 2, 3, 4]. The most intriguing facts are that the universe has not only the finite volume (to explain the Halley-de Chśeaux-Olbers paradox of the dark sky [5, 6, 7]) but also the finite life-time. The universe can be considered as one of ordinary physical objects described by differential equations of the General Relativity (GR) given in a definite frame of reference [8] with definite initial data and boundary conditions.

The cosmic evolution in the finite space-time are conventionally described in the framework of the homogeneous approximation [1, 2, 3, 4]. In the present paper, the cosmic evolution is considered in GR as a collective motion of metrics in the "field space". The geometry of this "field space" in GR was obtained by Borisov and Ogievetsky [9] in terms of Cartan forms [10, 11] as a geometry of the coset of the affine group $A(4)$ over the Lorentz one L. The Cartan method of constructing the nonlinear realization of the affine symmetry [10, 11], in particular the operation of the group summation formulated in [12] - [15], allows us to extend concepts of "collective" and "relative" coordinates, inertial motions (i.e., motions with constant canonical momenta) along "geodesic lines" over the coset $A(4)/L$.

These old concepts extended over the field space reveal in GR two alternative measurement standards: the absolute standard leading to the Friedmann-Robertson-Walker (FRW) cosmology [1, 2, 3, 4], and the relative standard leading to the conformal cosmology defined as the FRW cosmology expressed in terms of conformal quantities [16, 17]. Both the measurement standards are discussed on an equal footing, to compare them with observational data.

In the conformal cosmology we have varying masses and constant temperature [18] instead of the constant masses and varying volume and temperature in the FRW cosmology. In the framework of the conformal cosmology both the primordial element abundance [19] and the latest Supernova data on the redshift - luminosity-distance relation [20, 21, 22] are compatible with the stiff state [16]. We show here that the stiff state of the gravitation fields has a simple geometric meaning as the "inertial motion" of the universe along geodesic lines in the field coset $A(4)/L$.

The collective cosmic motion of the universe (inheriting the group of reparametrizations of the coordinate evolution parameter in GR) establishes a correspondence between GR and Special Relativity (SR). The direct GR/SR correspondence solves the problem of the time and energy in GR just as this problem was solved by Poincare and Einstein for a relativistic particle in 1904-05 [23, 24]. We use modern results [25, 26, 27, 28, 29, 30] on the dynamic description of the pure relativistic effects by the canonical Levi-Civita – type transformation [31, 32, 33, 34], to give the mathematically rigorous introduction of the geometrical time interval into quantum theory of the relativistic collective motion in the field space on the basis of the Dirac generalized mechanics [35]. The Levi-Civita canonical transformations convert the energy constraint into a new momentum; and the scale factor, into the visible (i.e., reparametrization - invariant) time interval measured by the watch of an observer in the universe. Simultaneously the Levi-Civita transformations convert the set of field variables into a new set of geometric variables with cosmic initial data.

It is well known that the quantization of cosmologi-

cal models in GR leads to the so-called Wheeler-DeWitt equations. The main problem is the lost of time in the Wheeler-DeWitt equations. Just this time is restored by the Levi-Civita transformations that link two evolution parameters and two wave functions of the universe in the field system and the geometric one. This Hamiltonian description of the cosmic evolution by two wave functions and their relation is the main difference of our approach from others (see references in [36]).

The GR/SR correspondence determines both the measurable energy density in the quantum field theory and particles as the holomorphic variables diagonalising this energy density. These variables and the mass-singularity of the massive vector bosons [37, 38]) in the Standard Models give theoretical basis of solving the problem of quantum cosmological creation of the visible matter in the universe [17, 39]

The content of the paper is the following: In Section 2 we give the description of the universe as the collective state of fields in the Einstein theory compatible with data of the observational cosmology. Section 3 is devoted to the reparametrization - invariant Hamiltonian description of the collective motion of the universe in both the classical theory and the quantum one. Section 4 is devoted to the cosmological creation of matter from vacuum.

II. GENERAL RELATIVITY AS A THEORY OF SPONTANEOUS BREAKDOWN OF AFFINE SYMMETRY

A. Gravity in terms of Cartan forms of the affine group

It is well known that the Einstein General Relativity (GR) is a gauge field theory. However, there is another deep analogue between gravitons in GR and pions in chiral dynamics based on nonlinear realizations of chiral symmetry $SU(2) \times SU(2)$. In 1974 Borisov and Ogievetsky [9] showed that GR was a theory of spontaneous breakdown of affine symmetry in the same way as chiral dynamics in QCD was a theory of spontaneous breakdown of chiral symmetry. In this theory ten gravitons are considered as the Goldstone particles.

A theory of Goldstone particles h [11, 12, 13, 14, 15] of nonlinear realizations of a semi-simple group G with subgroup of stability vacuum H is based on the Cartan forms ω and θ. These forms determine Lagrangian and covariant derivative with the help of a shift ω and rotations θ of repers in the factor-space (i.e. coset) $K = G/H$ defined by the finite transformations of group G through the equality

$$G^{-1}(h)\partial_\mu G(h) = i(\omega^m_\mu(h)X_m + \theta^n_\mu(h)Y_n) \qquad (1)$$

with the initial data

$$\omega^m_\mu(0) = 0, \qquad \theta^n_\mu(0) = 0 \ ,$$

where h are the group parameters identified with Goldstone particles, $Y_n(n = 1, ..., r)$ are the generators of the subgroup H, and $X_m(m = 1, ..., k)$ are the generators of the factor space G/H. In particular, in the exponential parametrization of group elements $G = \exp(ih^m X_m)$ the form $\omega(h)$ describes the transition along a geodesic line in the coset G/H. Making the change

$$G(h) \ \rightarrow \ G(\phi)G(h) \qquad (2)$$

in (1) and solving the equations for the Cartan forms with nonzero boundary conditions [12, 13, 14, 15]

$$\bar{\omega}^m_\mu(\phi, 0) = \omega^m_\mu(\phi), \qquad \bar{\theta}^n_\mu(\phi, 0) = \theta^n_\mu(\phi) \ , \qquad (3)$$

we get the extended Cartan forms $\bar{\omega}^m_\mu(\phi, h)$, $\bar{\theta}^n_\mu(\phi, h)$. These forms describe the transition from the point ϕ to point $\phi(+)h$ in the field space, and in the particular case of the exponential parametrization, the transition along a geodesic. The changes (2) and (3) are called the summation in the coset.

The definition of the geodesic lines and the group summation allow us to extend the concepts of "inertial motion", the collective and relative coordinate to the coset of the Goldstone fields, in particular in GR considered as a theory of nonlinear realization of the affine group $A(4)$ in the coset $K = A(4)/L$ with respect to the Lorentz subgroup.

The algebra of the affine group of all linear transformations of a four-dimensional space-time $A(4) = P_4 \times L(4, R)$ consists of generators of the Lorentz group $L_{\mu\nu}$, generators of proper affine transformations $R_{\mu\nu}$, and those of translation P_μ.

In the theory of nonlinear group representations the coordinates x_μ and ten Goldstone fields $h_{\mu\nu}$ (i.e., gravitons) are treated as parameters of the transformations in the factor space $A(4)/L$. Invariants under transformations with constant parameters are constructed with the help of Cartan forms

$$\begin{aligned} G^{-1}dG \ &= \ i[\omega^P_\mu(d)P_\mu + \frac{1}{2}\omega^R_{\mu\nu}(d)R_{\mu\nu} + \frac{1}{2}\omega^L_{\mu\nu}(d)L_{\mu\nu}] \ , \\ G \ &= \ \exp\{iP_\mu x_\mu\}\exp\{\frac{1}{2}iR_{\mu\nu}h_{\mu\nu}\} \ . \end{aligned} \qquad (4)$$

The form ω^R defines the covariant differential for the Goldstone field h; and the forms ω^P and ω^L, the covariant diferentiation of the external field Ψ transformed by the representation of the Lorentz group with the generators $L^\Psi_{\mu\nu}$. The Cartan forms are nothing but the Fock tedrads [40]

$$\omega^P_{\underline{\lambda}}(d) = e_{\underline{\lambda}\mu}dx^\mu \ , \qquad (5)$$

$$\omega^L_{\underline{\mu\nu}}(d) = \frac{1}{2}(\omega_{\underline{\mu\nu}}(d) - \omega_{\underline{\nu\mu}}(d)) = \omega_{[\underline{\nu\mu}]} \ , \qquad (6)$$

$$\omega^R_{\underline{\mu\nu}}(d) = \frac{1}{2}(\omega_{\underline{\mu\nu}}(d) + \omega_{\underline{\nu\mu}}(d)) = \omega_{[\underline{\nu\mu}]} \ , \qquad (7)$$

where

$$\omega_{\underline{\nu}\underline{\mu}} = de_{\underline{\nu}\sigma}(e^{-1})_{\sigma\underline{\mu}} \ , \tag{8}$$

and $e_{\underline{\nu}\sigma}(e^{-1})_{\sigma\underline{\mu}} = \delta_{\underline{\nu}\underline{\mu}} = \mathrm{diag}[1,-1,-1,-1]$. The Cartan forms allow us to realize the Fock separation of the Lorentz transformations from the general coordinate ones [40]. The Fock tetrads $e_{\underline{\lambda}\mu}$ belong to both the spaces: the Minkowski space marked by the underlined indices $\underline{\lambda}$ and the Riemannian space marked by the indices μ. The choice of the normal cordinates in the ten-dimentional space $h_{\underline{\nu}\ \underline{\mu}}$ corresponds to the exponential parametrization [9, 13]

$$e_{\underline{\nu}\sigma} = (\exp h)_{\underline{\nu}\sigma} \ . \tag{9}$$

It was shown in [13, 15] that the summation along geodesic lines $h(+)\phi$ corresponds to the choice of a group transformation in the form $G = G(\phi)G(h)$ or

$$\omega^P_{\underline{\lambda}}(d) = (\exp h \exp \phi)_{\underline{\lambda}\mu} dx^\mu \ . \tag{10}$$

The invariant elements of length and volume are constructed from the Cartan forms ω^P

$$\begin{aligned}(ds)^2 = \ & \omega^P_{\underline{\lambda}}(d)\omega^P_{\underline{\lambda}}(d) \equiv \omega^P_{\underline{0}}(d)\omega^P_{\underline{0}}(d) \\ & -\omega^P_{\underline{a}}(d)\omega^P_{\underline{a}}(d) \equiv g_{\mu\nu}dx^\mu dx^\nu\end{aligned} \tag{11}$$

$$\begin{aligned}dv = \ & \omega^P_{\underline{0}}(d)\omega^P_{\underline{1}}(d)\omega^P_{\underline{2}}(d)\omega^P_{\underline{3}}(d) \\ & = d^4x|-e| = d^4x\sqrt{-g} \ .\end{aligned} \tag{12}$$

The four-dimensional curvature $R = R_{\underline{\mu}\underline{\nu},\underline{\nu}\underline{\mu}}$, the Riemannian tensor

$$\begin{aligned}R_{\underline{\mu}\underline{\nu},\underline{\lambda}\underline{\rho}} = \ & (e^{-1})_{\sigma\underline{\lambda}}\partial_\sigma v_{\underline{\mu}\underline{\nu},\underline{\rho}} + v_{\underline{\mu}\underline{\nu},\underline{\gamma}} v_{\underline{\rho}\underline{\gamma},\underline{\lambda}} \\ & + v_{\underline{\mu}\underline{\gamma},\underline{\rho}} v_{\underline{\nu}\underline{\gamma},\underline{\lambda}} - (\underline{\lambda} \ \leftrightarrow \ \underline{\rho}) \ ,\end{aligned} \tag{13}$$

and the covariant differentation of the external field Ψ

$$D_\lambda\Psi = D\Psi/\omega^P_\lambda = [(e^{-1})_{\sigma\underline{\lambda}}\partial_\sigma + \frac{1}{2}iv_{\underline{\mu}\underline{\nu},\underline{\lambda}}L^\Psi_{\underline{\mu}\underline{\nu}}]\Psi \tag{14}$$

can be constructed from the Cartan forms [9], where $v_{\underline{\mu}\underline{\nu},\underline{\gamma}}$ is the sum of

$$\omega_{\underline{\nu}\underline{\mu},\underline{\gamma}} = (\partial_\lambda e_{\underline{\nu}\sigma})(e^{-1})_{\sigma\underline{\mu}}(e^{-1})_{\lambda\underline{\gamma}} \tag{15}$$

over all permutations of the indices with the sign $(+)$ for even ones; $(-)$, for odd.

In terms of these expressions the Einstein-Hilbert action added by the Standard Model one takes the form

$$\begin{aligned}& S_{\rm tot}[f,e|\varphi_0, M_{\rm Higgs}] = \\ & \int d^4x|-e|[-\frac{\varphi_0^2}{6}R(e) + \mathcal{L}_{\rm SM}(f,e)] \ ,\end{aligned} \tag{16}$$

where

$$\varphi_0^2 = M^2_{\rm Planck}\frac{3}{8\pi} \ . \tag{17}$$

One can find a direct analogue with the action of a relativistic particle in Special Relativity (SR)

$$S_{\rm SR} = -\frac{m}{2}\int\limits_{\tau_1}^{\tau_2} d\tau[\frac{\dot{X}^2_\mu}{e} + e] \ , \tag{18}$$

where the role of field variables is played by the coordinates of a particle. Instead of tetrads ("vier-bein") we have here "ein-bein" [23, 24]. The analogue of general coordinate transformations

$$x_\mu \Rightarrow \tilde{x}_\mu = \tilde{x}_\mu(x_0, x_1, x_2, x_3) \tag{19}$$

in SR is reparametrizations of the coordinate time

$$\tau \to \bar{\tau} = \bar{\tau}(\tau) \ . \tag{20}$$

That means that observable time is the invariant interval

$$ds = ed\tau = \bar{e}d\bar{\tau} \tag{21}$$

identified with the proper time measured by the watch in the comoving frame.

The principle of General Relativity means that coordinates of space-time $x^\mu = (x^0, x^2, x^2, x^3)$ and tetrads e in the action (16) are not observable. Observables are the Cartan forms, like in electrodynamics observables are gauge invariant tensions, but not the gauge - variant fields.

The Hilbert variational principle in terms of tetrads reproduces the classical Einstein equations

$$\frac{\varphi_0^2}{3}\sqrt{-g}\left[R^\nu_\mu(g) - \frac{1}{2}\delta^\nu_\mu R(g)\right] = \varepsilon^\nu_\mu \equiv \sqrt{-g}\ T^\nu_\mu \ , \tag{22}$$

where T^ν_μ is the matter energy - momentum tensor.

The problem is to solve equations (22) in terms of invariants of the group (19) in a definite *frame of reference*. The latter is defined as a set of physical instruments for measurement of *the initial data of independent variables* [8, 41, 42, 43, 44] (see Appendix A).

B. Cosmic evolution as a collective motion in the coset $A(4)/L$

Evolution of fields including the metrics in GR is studied in the kinemetric frame of reference [8] with the so-called Arnowitt-Deser-Misner (ADM) parametrization of the metric [41, 42]

$$\begin{aligned}(ds)^2 = \ & g_{\mu\nu}dx^\mu dx^\nu = (Ndx^0)^2 \\ & -{}^{(3)}g_{ij}\left(dx^i + N^i dx^0\right)\left(dx^j + N^j dx^0\right)\end{aligned} \tag{23}$$

The Hamiltonian description of GR in this frame is invariant with respect to reparametrization of the coordinate evolution parameter $x^0 \to \tilde{x}_0 = \tilde{x}_0(x^0)$. This invariance means that the coordinate evolution parameter x^0 cannot be measured, and we should point out a

measured evolution parameter among the set of the field variables to solve the problem of energy and time in the kinemetric frame of reference [28, 30, 49]. This measured evolution parameter is well known in cosmology as the cosmic scale factor.

In contrast to the conventional homogeneous approximation [1]-[4], we define the cosmic scale factor $a(x^0)$ as the functional of metrics in the form of an invariant spatial volume

$$\frac{1}{V_0}\int\limits_{V_0} d^3x\sqrt{{}^{(3)}g(x^0,x^i)} \equiv \frac{1}{V_0}\int\limits_{V_0} d^3x|{}^{(3)}e| = a^3(x^0), \quad (24)$$

where V_0 is the present-day value of the finite volume.

In contrast to the conventional Hamiltonian description of GR [41, 42, 43], we extract the cosmic scale factor as a collective coordinate in the space of metrics $g_{\mu\nu}$ considered as the coset A(4)/L [9]. We introduce the collective variable using the geometry of geodesic lines in the coset $A(4)/L$. The operation of adding along a geodesic line in terms of the normal coordinates in the field space $g_{\mu\nu}(h) = [\exp(2h)]_{\mu\nu}$ is defined by eq. (10) [13, 15]:

$$g_{\mu\nu}(h_{\rm coll.}(+)h_{\rm rel.}) = \{\exp(h_{\rm coll.})\exp(2h_{\rm rel.})\exp(h_{\rm coll.})\}_{\mu\nu} \ . \quad (25)$$

In this case, a collective motion of the volume $a(x^0)$ is separated from the relative metric $\bar g_{\mu\nu}(x^0,x^i)$ by the multiplication

$$g_{\mu\nu}(x^0,x^i) = \bar g_{\mu\nu}(x^0,x^i)a(x^0)^2 \quad (26)$$

corresponding to

$$N = \bar N a, \ g_{ij} = \bar g_{ij}a^2 \ .$$

The normal coordinate in the field space along a geodesic line is the Misner exponential parametrization of the scale factor [45]

$$a(x^0) = \exp X_0(x^0) \ . \quad (27)$$

Constant values of the canonical momentum of the Misner variable X_0 correspond to an *inertial motion* in the field space along a geodesic line.

Transformation (26) is a particular case of the Lichnerowicz conformal transformations [46] of all field variables $\{{}^{(n)}f\}$

$${}^{(n)}f(x^0,x^i) = {}^{(n)}\bar f(x^0,x^i)a(x^0)^n \quad (28)$$

with a conformal weight n, including the metric as a tensor field with the conformal weight $n=2$ [47]. We suggest that each field contributes to the cosmic evolution of the universe in line with the Lichnerowicz conformal transformations (28). The auxiliary variable can be removed by the constraint of the constant spatial volume in the relative field space $\bar g_{\mu\nu}$ that follows from eqs. (24), (26)

$$\int\limits_{V_0} d^3x|{}^{(3)}\bar e(x^0,x^i)| \equiv V[\bar e] = V_0 \ . \quad (29)$$

To identify this *collective variable* with the homogeneous cosmic scale factor in observational cosmology, we should verify that the exact equations of $a(x^0)$ averaged over the invariant three-dimensional volume in our theory coincide with the equations of homogeneous cosmic scale factor in the standard cosmology where the concept of the cosmic evolution of the universe is formulated.

C. Exact equations of the cosmic evolution

To find the Einstein action with the collective motion and corresponding Einstein equations, we use the well-known formula of conformal transformations (26) of a four-dimensional curvature

$$\sqrt{-g}\frac{\varphi_0^2}{6}R(g) = \sqrt{-\bar g}\frac{\varphi^2}{6}R(\bar g) - \varphi\partial_\mu\left[\sqrt{-\bar g}\bar g^{\mu\nu}\partial_\nu\varphi\right], \quad (30)$$

where $\varphi(x^0)$ is the dynamic Planck mass defined as the product of the Planck mass and the cosmic scale factor

$$\varphi(x^0) = a(x^0)\varphi_0 \qquad \left(\varphi_0 = M_{\rm Planck}\sqrt{\frac{3}{8\pi}}\right). \quad (31)$$

This formula leads to the Einstein action

$$\begin{aligned} S_{GR}[e|\varphi_0] = \ & S_{GR}[\bar e|\varphi] \\ & + \int\limits_{x_1^0}^{x_2^0} dx^0 \int\limits_{V_0} d^3x\varphi\frac{d}{dx^0}\left(|{}^{(3)}\bar e|\frac{d\varphi}{\bar N dx^0}\right) \\ & + \int\limits_{x_1^0}^{x_2^0} dx^0\Lambda(x^0)\left[V[\bar e]-V_0\right] \ , \end{aligned} \quad (32)$$

where the Lagrangian factor $\Lambda(x^0)$ provides the conservation of the volume (29) of local excitations;

$$S_{\rm GR}[\bar e|\varphi] = -\int d^4x\sqrt{-\bar g}\frac{\varphi^2}{6}R(\bar g)$$

$$= \int\limits_{x_1^0}^{x_2^0} dx^0\int\limits_{V_0} d^3x\left[\mathtt{K}(\bar e|\varphi) - \mathtt{P}(\bar e|\varphi) + \mathtt{S}(\bar e|\varphi)\right] \quad (33)$$

is the standard ADM action in GR with the relative metric $\bar g$ and the running Planck mass where

$$\mathtt{K}(\bar e|\varphi) = \frac{\varphi^2|{}^{(3)}\bar e|}{24N}\left[\bar\pi_{\underline{ab}}\bar\pi_{\underline{ab}} - \bar\pi_{\underline{bb}}\bar\pi_{\underline{aa}}\right] \quad (34)$$

is the kinetic term with the external form

$$\pi_{\underline{ab}} = \frac{1}{2}[(D_0e)_{\underline{a}i}(e^{-1})_{i\underline{b}} + (\underline{a} \ \leftrightarrow \ \underline{b})]) \ ,$$

$$(D_0e)_{\underline{a}i} = \left(\partial_0 - N^l\partial_l\right)e_{\underline{a}i} - e_{\underline{a}l}\partial_iN^l \ , \quad (35)$$

$$\mathtt{P}(\bar{e}|\varphi)=\frac{\varphi^2\bar{N}|^{(3)}\bar{e}|}{6}\ \ {}^{(3)}R({}^{(3)}\bar{g}) \tag{36}$$

is the potential term, and

$$\begin{aligned}\mathtt{S}(\bar{e}|\varphi)= &\ \frac{\varphi^2}{6}\left(\partial_0-\partial_k N^k\right)\left(|^{(3)}\bar{e}|\frac{\bar{\pi}_{\underline{aa}}}{\bar{N}}\right)\\ &-\frac{\varphi^2}{3}\partial_i\left(|^{(3)}\bar{e}|^{(3)}\bar{g}^{ij}\partial_j\bar{N}\right)\end{aligned} \tag{37}$$

are the standard ADM "surface terms" contributing to the equations of motion due to the time dependence of the dynamic Planck mass φ. This contribution describes the interference between the local relative excitations and the collective one.

The action of collective motion allows us to define the global lapse function

$$\frac{1}{\bar{N}_0(x^0)}=\frac{1}{V_0}\int\limits_{V_0}d^3x\frac{|^{(3)}\bar{e}|}{\bar{N}}. \tag{38}$$

and the gauge - invariant *world geometric time*

$$d\eta=\bar{N}_0(x^0)dx^0=\tilde{\bar{N}}_0(\tilde{x}^0)d\tilde{x}^0\ . \tag{39}$$

In terms of *world geometric time* the variation of the total action (16) with respect to the lapse function and determinant of spatial metric leads to the equations

$$\bar{N}\frac{\delta S_{\rm tot}}{\delta\bar{N}}=0\ \Longrightarrow\ \frac{\varphi'^2}{\mathcal{N}}=\mathcal{N}\bar{T}^0_0, \tag{40}$$

$$\bar{g}^{ij}\frac{\delta S_{\rm tot}}{\delta\bar{g}^{ij}}=0\ \Longrightarrow\ \frac{2(\varphi^2)''-3\varphi'^2}{\mathcal{N}}+3\Lambda=\mathcal{N}\bar{T}^k_k, \tag{41}$$

$$\frac{\delta S_{\rm tot}}{\delta\bar{N}^k}=0\ \Longrightarrow\ \bar{T}^0_k=0, \tag{42}$$

$$\bar{g}^{ki}\frac{\delta S_{\rm tot}}{\delta\bar{g}^{ij}}=0\ \Longrightarrow\ \bar{T}^i_k=0\ \ (i\neq k), \tag{43}$$

where $f'=df/d\eta, \mathcal{N}=\bar{N}/\bar{N}_0$, and $\bar{T}^\nu_\mu=T^\nu_\mu-\varphi^2/3(R^\nu_\mu-1/2\delta^\nu_\mu R)$ are the total components of the local energy-momentum tensor

$$|^{(3)}\bar{e}|\mathcal{N}\bar{T}^0_0=\mathtt{K}(\bar{e}|\varphi)+\mathtt{P}(\bar{e}|\varphi)+\varepsilon^0_0\equiv\varepsilon^0_{0({\rm tot})}\ , \tag{44}$$

$$3\mathtt{K}(\bar{e}|\varphi)-\mathtt{P}(\bar{e}|\varphi)+2\mathtt{S}(\bar{e}|\varphi)+\varepsilon^k_k\equiv\varepsilon^k_{k({\rm tot})} \tag{45}$$

($\bar{T}^\nu_\mu=0$ is equal to zero, if the cosmic evolution is absent $\varphi(x^0)\equiv\varphi_0$). These equations contain the collective motion of the cosmic evolution that can be extracted by integration of these equations over the spatial volume. As a result we get

$$\frac{1}{V_0}\int\limits_{V_0}d^3x\bar{N}\frac{\delta S_{\rm tot}}{\delta\bar{N}}=0\ \Longrightarrow\ \varphi'^2=\rho_{\rm tot}, \tag{46}$$

$$\begin{aligned}\frac{1}{V_0}\int\limits_{V_0}d^3x\bar{g}^{ij}\frac{\delta S_{\rm tot}}{\delta\bar{g}^{ij}}=0\ \Longrightarrow\ &(\varphi^2)''-3\varphi'^2+3\Lambda=\\ &-3p_{\rm tot}\ ;\end{aligned} \tag{47}$$

here we introduce the Gibbs averaging

$$\rho_{\rm tot}=\frac{1}{V_0}\int d^3x\varepsilon^0_{0({\rm tot})}\ ,$$

$$3p_{\rm tot}=\frac{1}{V_0}\int d^3x\varepsilon^k_{k({\rm tot})}.$$

These equations are accompanied by the equations of collective variables

$$\varphi\frac{\delta S_{\rm tot}}{\delta\varphi}=0\ \Longrightarrow\ 2\varphi\varphi''=\rho_{\rm tot}-3p_{\rm tot}, \tag{48}$$

$$\frac{\delta S_{\rm tot}}{\delta\Lambda}=0\ \Longrightarrow\ V[\bar{g}]-V_0=0. \tag{49}$$

The combination of eqs. (46), (47), and (48) leads to $\Lambda=0$.

In this case, the exact equations (46) and (47) in *relative field space* for the collective variable completely coincide with the conformal version of the equations of the Friedmann-Robertson-Walker (FRW) cosmology in the homogeneous approximation [1]-[4]

$$\boxed{\varphi_0^2a'^2=\rho_{\rm tot}\ ;\quad \varphi_0^2\left[3a'^2-(a^2)''\right]=3p_{\rm tot}\ ,} \tag{50}$$

where $\rho_{\rm tot}$ and $p_{\rm tot}$ are the total density and the total pressure. Transition to physical values (time t, distance l, density $\rho_{\rm F}$) of the FRW cosmology is carried out with the help of conformal transformations

$$t\ =\ \int_0^\eta d\bar{\eta}a(\bar{\eta}), \tag{51}$$

$$l\ =\ a(\eta)r,\quad r=\sqrt{x_1^2+x_2^2+x_3^2} \tag{52}$$

$$\rho_{\rm F}(a)\ =\ \frac{\rho_{\rm tot}(a)}{a^4}\ . \tag{53}$$

In the terms of the FRW cosmology the equation of evolution (50) takes the conventional form

$$\boxed{\varphi_0^2\left(\frac{da}{dt}\right)^2=\rho_{\rm F}(a)\ ;\quad \varphi_0^2\left[\left(\frac{da}{dt}\right)^2+a\frac{d^2a}{dt^2}\right]=-3p_{\rm F}.} \tag{54}$$

In addition, by substituting eqs. (46) and (47) into (40) and (41), we obtain the equations of the local excitations

$$\frac{|^{(3)}\bar{e}|}{\mathcal{N}V_0}\int\limits_{V_0}d^3x\varepsilon^0_{0({\rm tot})}=\varepsilon^0_{0({\rm tot})}\ , \tag{55}$$

$$\frac{|^{(3)}\bar{e}|}{\mathcal{N}V_0}\int\limits_{V_0} d^3x\varepsilon^k_{k(\mathrm{tot})}=\varepsilon^k_{k(\mathrm{tot})}\ ,$$

where $\varepsilon^0_{0(\mathrm{tot})}$, $\varepsilon^k_{k(\mathrm{tot})}$ are given by eqs. (44) and (45).

These local equations are compatible with the cosmological equations (46), (47), and in the infinite volume limit the local equations (55) coincide with the ordinary Einstein equations in the Riemannian space-time. It was shown that the cosmic evolution changed the Newton law and the black-hole solution in the Early Universe [17, 49].

D. Measurement standards

It is worth reminding that the concept of measurable quantities in the field theory is no less important than the equations of the theory. J.C. Maxwell wrote: "The most important aspect of any phenomenon from mathematical point of view is that of a measurable quantity. I shall therefore consider electrical phenomena chiefly with a view to their measurement, describing the methods of measurement, and defining the standards on which they depend"[48].

Suppose that nature selects itself both the theory and standards of measurement, and the aim of observation is to reveal not only initial data, but also these measurement standards. In particular, one of the central concepts of the modern cosmology is the concept of the scale defined as a functional of spatial volume in GR [49]. If expanding volume of the universe means the expansion of "all its lengths", we should specify whether the measurement standard of length expands. Here there are two possibilities: the first, the absolute measurement standard does not expand; and the second, the relative measurement standard expands together with the universe.

Until the present time the first possibility was mainly considered in cosmology. The second possibility means that we have no absolute instruments to measure absolute values in the universe. We can measure only a ratio of values which does not depend on the spatial scale factor. The relative measurement standard transforms the spatial scale of the intervals of lengths into the scale of masses which permanently grow.

As we have shown in the previous section the universe evolution as a collective motion of the spatial volume in the field "space"

$$(\varphi,\ \bar{g}_{\mu\nu},\bar{f}...)\equiv(\varphi,\ \bar{F})$$

reproduces the equations of a conformal version of the FRW cosmology, if the homogeneous approximation is changed by averaging the local density and pressure. To obtain the equation of the standard cosmology, it is sufficient to make the reverse conformal transformations (28) of the *relative* quantities ${}^{(n)}\bar{F}$ into the quantities of *absolute field space* ${}^{(n)}F={}^{(n)}\bar{F}a^n$. The FRW cosmology supposes that our instruments measure absolute fields ${}^{(n)}F$ and a "absolute" interval of the Riemannian space

$$(ds)^2=g_{\alpha\beta}dx^\alpha dx^\beta\ . \qquad (56)$$

The "relative" point of view supposes that our instruments measure relative fields ${}^{(n)}\bar{F}$ and a "relative" interval of the Riemannian space

$$(d\bar{s})^2=\bar{g}_{\alpha\beta}dx^\alpha dx^\beta\equiv\frac{(ds)^2}{a^2}\ . \qquad (57)$$

E. The GR/SR Hamiltonian correspondence

The conventional Hamiltonian formulation in terms of Dirac theory of constrained systems [35] is given by the action

$$S_{\mathrm{GR}}=\int dx^0\left\{\left[\int d^3x\sum_{i,\underline{a}}P_{i\underline{a}}\partial_0\bar{e}^i_{\underline{a}}\right]-H_{\mathrm{tot}}[\bar{e}|\varphi]\right\}\ , \qquad (58)$$

where

$$H_{\mathrm{tot}}[e|\varphi_0]=\int d^3x\left[N\mathcal{H}-N^k\mathcal{P}_k+C_0P_g+C_{\underline{b}}f_{\underline{b}}(e)-\mathtt{S}\right] \qquad (59)$$

is the Hamiltonian, N, N^k, C, C^k are the Lagrangian multipliers for the first class constraints for densities of energy $\mathcal{H}=0$ and momenta $\mathcal{P}_k=0$, and second class ones that are the Dirac conditions of transverseness $f_{\underline{b}}(e)=0$ and the minimum embedding of three-dimensional hypersurface into the four-dimensional Riemannian space-time with the zero momentum of spatial metric determinant $P_g=0$ [41]. The latter means that the second (external) form $\pi_{\underline{a},\underline{a}}$ is equal to zero that contradicts to the cosmic evolution with the nonzero Hubble parameter proportional to $\pi_{\underline{a},\underline{a}}$ (see Appendix A, eq. (A.33)).

On the other hand, the conventional description of the cosmic evolution keeps only the homogeneous part of the second form neglecting all local excitations $H_{\mathrm{tot}}=0$.

To include the cosmic evolution into field theory of the local excitations, we have define this evolution [26, 28] as the collective variable $\varphi(x^0)=a(x^0)\varphi_0$ by eq. (24).

The Einstein theory after the separation of the collective motion takes the form

$$S_{\mathrm{GR}}[e|\varphi_0]=S_{\mathrm{GR}}[\bar{e}|\varphi]+S_{\mathrm{interference}}+S_{\mathrm{collective}}\ , \qquad (60)$$

where the first term coincides with the initial Einstein action in terms of relative fields and *dynamic evolution parameter* φ instead of the Planck mass φ_0; the second term

$$S_{\mathrm{interference}}=-\frac{1}{6}\int\limits_{x^0_1}^{x^0_2}dx^0\partial_0(\varphi^2)\int\limits_{V_0}d^3x\frac{\bar{\pi}_{\underline{a}\underline{a}}}{\bar{N}} \qquad (61)$$

goes from the first one in eq. (37) in the relative metric. This term describes an interaction of the collective and relative variables. The third term

$$S_{\rm collective} = -V_0 \int\limits_{x^0_1}^{x^0_2} dx^0 \frac{(\partial_0\varphi)^2}{\bar N_0} \qquad (62)$$

is the collective motion of the universe.

The interference of the collective and relative variables disappears, if we impose the Dirac condition [41] of the minimal embedding of the three-dimensional hypersurface into the four-dimensional space-time in the relative space

$$\bar\pi_{\underline{aa}} = 0 \ . \qquad (63)$$

The minimal embedding removes not only the interference of the collective motion with local excitations, but also all local excitations with the negative norm [43].

In the case of the minimal embedding the Hamiltonian form of the Einstein action (58) with the collective motion takes the form

$$S_{\rm GR}[e|\varphi_0] = \int dx^0 \left[\int d^3x \sum_{i,\underline{a}} P_{i\underline{a}} \partial_0 \bar e^i_{\underline{a}} - H_{\rm tot}[\bar e|\varphi] - P_\varphi \partial_0\varphi + \bar N_0 \frac{P^2_\varphi}{4V_0} + \Lambda(\bar V[\bar e] - V_0) \right] \ ; \qquad (64)$$

here the relative Hamiltonian $H_{\rm tot}[\bar e|\varphi]$ is the conventional one (59) where all fields are changed by the relative one $\bar e^i_{\underline{a}}, \bar N$ and the Planck mass φ_0 is changed by the running Planck mass as *dynamic evolution parameter* φ, $\bar N_0[\bar e, \bar N]$ and $\bar V[\bar e]$ are considered as functionals given by eqs. (29) and (38)

$$\frac{1}{\bar N_0} = \frac{1}{V_0} \int\limits_{V_0} d^3x \frac{|^{(3)}\bar e|}{\bar N}, \qquad \bar V[\bar e] = \int\limits_{V_0} d^3x |^{(3)}\bar e| \ . \qquad (65)$$

The GR with the collective motion is the direct field generalization of SR with two time-like variables (the geometric interval $d\eta = N_0 dx^0$ and *dynamic evolution parameter* φ) and two wave functions. We have one to one correspondence between SR and GR [27, 28], i.e., their proper times

$$ds = e d\tau \qquad \Longleftrightarrow \qquad d\eta = \bar N_0 dx^0 \ , \qquad (66)$$

their world spaces

$$X_0, \ X_i \qquad \Longleftrightarrow \qquad \varphi, \ \bar e^i_{\underline{a}} \ , \qquad (67)$$

their energies

$$P_0 = \pm\sqrt{P_i^2 + m^2} \ \Longleftrightarrow \ P_\varphi = \pm 2\sqrt{V_0 H_{\rm tot}[\bar e|\varphi]}, \quad (68)$$

and their two-time relations in the differential form

$$\frac{dX_0}{ds} = \pm\frac{\sqrt{P_i^2+m^2}}{m} \ \Longleftrightarrow \ \frac{d\varphi}{d\eta} = \pm\sqrt{\rho_{\rm tot}(\varphi)} \qquad (69)$$

and in the integral forms

$$s(X_0) = \pm \frac{m}{\sqrt{P_i^2+m^2}} X_0 \ \Longleftrightarrow \eta(\varphi_0, \varphi_I) = \pm \int\limits_{\varphi_I}^{\varphi_0} \frac{d\varphi}{\sqrt{\rho_{\rm tot}(\varphi)}} \ . \qquad (70)$$

Recall that in SR eq. (70) can be treated as the Lorentz transformation of the rest frame with time X_0 into the comoving one with the proper time s. Similarly, in GR the relation $\eta(\varphi)$ defined by eq. (70) is treated as a canonical transformation [28]. This GR/SR correspondence (66)-(70) allows us to solve the problem of time and energy in GR like Poincare and Einstein [23, 24] had solved it in SR. They identified the time with one of variables in the world space. The similar String/SR correspondence was considered in papers [29, 30, 49].

F. Cosmic evolution as an inertial motion in the coset $A(4)/L$

The cosmic collective motion as the dynamics of the scale factor a can be separated in any theory, in particular, in the unified theory considered as the sum of GR and the Standard Model

$$S_{\rm tot}[\{F\}|\{M\}] = S_{\rm GR}[e|M_{\rm Planck}] + S_{\rm SM}[e,\{f\}|M_{\rm Higgs}] \quad (71)$$

with a set of fields $\{F\} = e, \{f\}$ and a set of massive parameters $\{M\}$ including the Higgs mass. This separation is fulfilled by the Lichnerowicz transformations of fields with the conformal weight n ${}^{(n)}F = {}^{(n)}\bar{F}a^n$ (28). As the result, the action (71) takes the form

$$S_{\rm tot}[\{F\}|\{M\}] = S_{\rm tot}[\{\bar{F}\}|\{Ma\}] + S_{\rm collective}[a,\bar{N}_0]\ , \quad (72)$$

where

$$S_{\rm collective}[a,\bar{N}_0] = -\gamma \int\limits_{x^0(I)}^{x^0(0)} dx^0 \left[\frac{(\partial_0 a(x^0))^2}{\bar{N}_0}\right]\ , \quad (73)$$

where $\gamma = V_0\varphi_0^2$. To make the analogue with a relativistic particle more transparent, we can pass to the normal coordinates in the field space along a geodesic line that corresponds to the choice of the Misner parametrization of the evolution parameter $X_0 = \log a$ in the field space (27) and the Misner lapse function

$$a = e^{X_0}; \qquad e_0 = \bar{N}_0 e^{-2X_0}\ . \quad (74)$$

Then, instead of the action (73) we get

$$S_{\rm collective}[X_0,N_0] = -\gamma \int\limits_{x_1^0}^{x_2^0} dx^0 \left[\frac{(\partial_0 X_0)^2}{e_0}\right]\ . \quad (75)$$

We call the collective motion along the geodesic line *inertial*, if the canonical momentum of the field evolution parameter X_0

$$P_0 = -2\gamma\frac{(\partial_0 X_0)}{e_0} \quad (76)$$

is a constant

$$\frac{dP_0}{dx^0} = -2\gamma\frac{d}{dx^0}\left[\frac{(\partial_0 X_0)}{e_0}\right] = 0 \implies \frac{dX_0}{e_0 dx^0} = H_0\ . \quad (77)$$

The solution of this equation is expressed in terms of the invariant "Misner time" $d\Omega = e_0 dx^0$

$$X_0(\Omega) = H_0 \int^{x^0} e_0(\bar{x}^0)d\bar{x}^0 = H_0\Omega(\eta)\ . \quad (78)$$

The dependence $\Omega(\eta)$ on the conformal time $d\eta = \bar{N}_0 dx^0$ (39) follows from eq. (74)

$$e_0 dx^0 = d\Omega = e^{-2X_0(\Omega)}d\eta\ . \quad (79)$$

The solution of this equation takes the form

$$X_0(\Omega) = H_0\Omega(\eta) \equiv \frac{1}{2}\log[1+2H_0(\eta-\eta_0)]\ . \quad (80)$$

In this case, the scale factor $a = \exp(X_0)$ is proportional to the square root of the conformal time (39) $d\eta = \bar{N}_0 dx^0$

$$a^2(\eta) = [1+2H_0(\eta-\eta_0)]\ , \quad (81)$$

where η_0 is the present-day value of time $a(\eta_0) = 1$. This result follows directly from the equations of motion (50) if the pressure is equal to the density

$$p_{\rm tot}(a) = \rho_{\rm tot}(a) = \frac{\varphi_0^2 H_0^2}{a^2} \quad (82)$$

and these equations reduce to

$$(a^2)'' = 0\ . \quad (83)$$

Thus, the *inertial* cosmic motion corresponds to the so-called stiff state (82). As it is known from the observational cosmology, the standard matter $S_{\rm tot}[\{\bar{F}\}|\{Me^{X_0}\}]$ gives a small contribution to the cosmic evolution.

Therefore, we propose that the equation of the stiff state is described by an additional action $S_I[e_0]$ so that the complete action takes the form

$$\begin{aligned} S_{\rm tot}[\{F\}|\{M\}] + S_I[e_0] = \ & S_{\rm tot}[\{\bar{F}\}|\{M_I e^{X_0}\}] \\ & + S_{\rm collective}[X_0,e_0] \\ & + S_I[e_0]\ . \end{aligned} \quad (84)$$

If we neglect the first term in (84), the additional action $S_I[e_0]$ leads to the inertial motion of the universe along geodesic with the density (82)

$$\begin{aligned} S_{\rm universe} &= S_{\rm collective}[X_0,N_0] + S_I[e_0] = \\ &- \gamma \int\limits_{x_1^0}^{x_2^0} dx^0 \left[\frac{(\partial_0 X_0)^2}{e_0} + e_0 H_0^2\right]\ . \end{aligned} \quad (85)$$

This action describes the relativistic universe in which the problem of energy is solved like in Special Relativity (18).

Thus, there are two differences of the cosmic motion in the coset $A(4)/L$ from the standard cosmology in the FRW metrics. These are the reparametrization invariance and the relative measurement standard in the coset which leads to the *conformal cosmology* with a constant volume of the flat space

$$d\bar{s}^2 = d\eta^2 - dx^i dx^i, \qquad r^2 = x^i x^i \equiv (x^1)^2+(x^2)^2+(x^3)^2$$

and varying masses $\bar{M}(\eta) = Ma(\eta)$ defined by eq. (81). Therefore, the spectrum of atoms is described by the Schrödinger equation

$$\left[\frac{\hat{p}^2}{2m_0 a(\eta)} - \left(\frac{\alpha}{r} + E(\eta)\right)\right]\Psi_A = 0 \ . \tag{86}$$

It is easy to check that the exact solution of this equation is expressed throw the solution E_0 of a similar Schrödinger equation with constant masses m_0 at $a(\eta_0) = 1$

$$E(\eta) = a(\eta)E_0 \equiv \frac{E_0}{z(d)+1} \ , \qquad E_0 = -\frac{m_0\alpha^2}{n^2} \ , \tag{87}$$

where $z(d)$ is a redshift of the spectral lines of atoms at the coordinate distance $d/c = \eta_0 - \eta$, and η_0 is the present-day value of the geometric (conformal) time.

G. Conformal cosmology and SN data

This type of conformal cosmology was developed by Hoyle and Narlikar [18]. A red photon emitted by an atom at a star two billion years (in terms of η) remembers the size of this atom, and after two billion years this photon is compared with a photon of the standard atom at the Earth that became blue due to the evolution of all masses. The redshift-coordinate distance relation is defined by the formula of the standard cosmology (87)

$$z(d) = \frac{a(\eta_0)}{a(\eta_0 - d/c)} - 1 \ , \qquad a(\eta_0) = 1 \tag{88}$$

(where d is the coordinate distance to an object) because the description of the conformal - invariant photons does not depend on the standard of measurements.

In the case of the inertial motion (81) this redshift - distance relation takes the form

$$z(d) = \frac{1}{(1 + 2H_0 d/c)^{1/2}} - 1 \ . \tag{89}$$

It results in the following simple relation

$$d(z) = \frac{c}{2H_0}\left[1 - \frac{1}{(1+z)^2}\right] \ . \tag{90}$$

The redshift - luminosity distance relation is determined by the formula $\ell_{\rm luminocity}(z) = (1+z)^2 d(z)$. The factor $(1+z)^2$ comes from the evolution of the angular size of the light cone of absorbed photons [16]. Since measurable distances in the conformal cosmology are the coordinate ones, we lose the factor $(1+z)^{-1}$ that was in the standard cosmology due to the expansion of the universe. Finally we obtain the redshift-luminosity distance relation

$$\ell_{\rm luminocity}(z) = (1+z)^2 d(z) = \frac{c}{H_0}\left[z + \frac{z^2}{2}\right] \tag{91}$$

as the consequence of the "inertial motion" of the universe along the geodesic line of the field *space* (i.e., the stiff state of dark energy with the most singular behaviour).

It has been shown in paper [16] that this relation does not contradict the latest Supernova data [20, 21, 22]. Among the CC models the pure stiff state of dark energy gives the best description and it is equivalent to the SC fit up to the distance of SN1997ff.

H. Primordial element abundance

In the considered model of the conformal cosmology the temperature is a constant. In the conformal cosmology we have the mass history [16]

$$m_{\rm era}(z_{\rm era}) = \frac{m_{\rm era}(0)}{(1 + z_{\rm era})} \tag{92}$$

with the constant temperature $T = 2.73\,{\rm K} = 2.35\times10^{-13}$ GeV where $m_{\rm era}(0)$ is characteristic energy (mass) of the era of the universe evolution, which begins at the redshift $z_{\rm era}$.

Eq. (92) has an important consequence that all physical processes, which concern the chemical composition of the universe and depend basically on the Boltzmann factors with the argument (m/T), cannot distinguish between the conformal cosmology $(\bar{m}/\bar{T})$ in the stiff state with the square root dynamics (81) (in the "relative" standard)

$$a(\eta) = \sqrt{1 + 2H_0(\eta - \eta_0)} \tag{93}$$

and the FRW cosmology in the radiation state with the same square root dynamics (in the "absolute" standard) due to the relations

$$\frac{\bar{m}(z)}{\bar{T}(0)} = \frac{m(z=0)}{(1+z)T(z=0)} = \frac{m(0)}{T(z)} \ . \tag{94}$$

From this formula it is clear that the z-history of masses with invariant temperatures in the stiff state of conformal cosmology is equivalent to the z-history of temperatures with invariant masses in the radiation stage of the standard cosmology. We expect, therefore, that the conformal cosmology allows us to keep the scenarios developed in the standard cosmology in the radiation stage for, e.g. the neutron-proton ratio, primordial element abundance, and the appearance of CMB radiation with the temperature $2.7K$.

Instead of the z-dependence of the temperature in an *expanding universe* with constant masses in the standard cosmology, in conformal cosmology, we have the z-history of masses in a *static universe* with an almost constant temperature of the photon background (with the same argument of the Boltzmann factors).

Thus, the "relative" cosmology [16] leads to the simplest Cold Universe Scenario with the fundamental parameter of the CMB temperature $2.7K$. In this Cold

Scenario the single stiff state (treated as a primordial inertial motion along a geodesic line of the field space) describes two last eras: 2) the chemical evolution, and 3) the present-day stage. One can suppose that the same inertial motion (79), (81), and (82) describes also the primordial era of creation of the universe and creation of matter at the beginning of the universe. The description of the cosmological creation of matter requires a complete and consistent solution of problems of cosmic singularity $a = 0$, cosmic initial data, and a positive arrow of the geometric time at the level of the quantum cosmic mechanics. We consider the solution of all these problems using the simplest case of the inertial motion.

III. HAMILTONIAN DESCRIPTION OF QUANTUM RELATIVISTIC UNIVERSE

A. Hamiltonian formalism of cosmic inertial motion

The action of a relativistic universe (85) in terms of the canonical momentum (76) of the field evolution parameter X_0 takes the form [45, 50]

$$S_{universe} = \int\limits_{x_1^0}^{x_2^0} dx^0 \left[-P_0 \partial_0 X_0 + \gamma e_0 \left(\frac{P_0^2}{4\gamma^2} - H_0^2 \right) \right] . \tag{95}$$

The Hamiltonian form (95) of the action (85) is well-known as the Dirac generalized mechanics [35] of relativistic systems, in particular, a relativistic particle (18). In this action (95) one of components of the metric in the Einstein theory plays the role of the time-like "coordinate of the Minkowskian space-time". Therefore, we call the variable X_0 the *world field time*. Whereas, the Misner *geometric* interval (74)

$$d\Omega = e_0 dx^0 \qquad \Rightarrow \qquad \Omega = \int\limits_{x_1^0}^{x_2^0} dx^0 e_0(x^0) \tag{96}$$

is connected by eq. (80) with the conformal time (39) measured by the watch of an observer. This *geometric* interval is invariant with respect to reparametrizations of the coordinate evolution parameter

$$x^0 \ \rightarrow \ \tilde{x}^0 = \tilde{x}^0(x^0) \ , \quad e_0 \ \rightarrow \ \tilde{e}_0 \ . \tag{97}$$

Classical equations of the generalized Hamiltonian system (95) split into the equation of motion

$$\frac{dX_0}{e_0 dx^0} = \frac{dX_0}{d\Omega} = \frac{P_0}{2\gamma}, \tag{98}$$

$$\frac{dP_0}{e_0 dx^0} = \frac{dP_0}{d\Omega} = 0$$

and the energy constraint

$$\frac{P_0^2}{4\gamma^2} - H_0^2 = 0 \ . \tag{99}$$

One can see that solutions of these equations are expressed in terms of the invariant geometric interval (96)

$$X_0(\Omega) = X_0(0) + \frac{P_0}{2\gamma}\Omega \ , \tag{100}$$

$$P_0 = \pm 2\gamma H_0 \ . \tag{101}$$

We have seen above that these solutions can describe the classical evolution of the universe. However, these solutions do not allow us to determine the dependence of the lapse-function $e_0(x^0)$ on the coordinate parameter x^0. There is ambiguity in the lapse function $e_0(x^0)$, as it can be an arbitrary function. On the other hand, we need the lapse function as the variation of the action with respect to it leads to the energy constraint (101). The problem arises whether we should fix $e_0(x^0)$ or not. Here, we face different mathematical and physical statements of the problem of the description of a constrained system.

B. Creation of quantum universe in *world field space*

The **mathematical statement of the problem is to give a logically consistent description** of the considered relativistic system in its classical and quantum versions. This description includes solutions to all functions. If there is ambiguity, we have to fix it, for example,

$$e_0(x^0) = 1 \tag{102}$$

In this case, x^0 is identified with the measurable time; and its Hamiltonian (that coincides with the constraint (99)), with a physical Hamiltonian. It seems that the problem of the classical description is completely solved. The Hamiltonian of evolution with respect to the "time" x^0 is identified with the constraint (99). In the corresponding quantum theory (where $\hat{P}_0 = id/dX_0$ is the operator), the wave function of the universe satisfies the quantum version of the constraint (99)

$$\left(\frac{\hat{P}_0^2}{4\gamma^2} - H_0^2 \right) \Psi_{wdw}(X_0) = 0 \ , \tag{103}$$

This quantization for a universe is well known as the Wheeler-DeWitt (WDW) one. The WDW wave function as the amplitude of transition from an initial scale factor $a_I = e^{X_{0I}}$ to a running scale factor $a_0 = e^{X_0}$ is decomposed over eigenvalues $P_0 = \pm 2\gamma H_0$

$$\begin{aligned} \Psi_{\rm wdw}(X_0|X_{0I}) = A^+ \Psi_+(X_0|X_{0I}) \theta(X_0 - X_{0I}) \\ + A^- \Psi_-(X_0|X_{0I}) \theta(X_{0I} - X_0), \end{aligned} \tag{104}$$

where the wave functions $\Psi_\pm$ satisfy the equations

$$\pm \frac{d}{idX_0}\Psi_\pm(X_0|X_{0I}) = H_0\Psi_\pm(X_0|X_{0I}) \ . \qquad (105)$$

Solutions of these equations

$$\Psi_\pm(X_0|X_{0I}) = \theta(\pm P_0)\exp\{-iP_0(X_0 - X_{0I})\} \ , \qquad (106)$$

$$P_0 = \pm 2\gamma H_0$$

depend on the cosmic initial data $a_I = e^{X_{0I}}$ (101) considered as an input parameter of the theory. The coefficient A^+ in the second quantization

$$\left[A^-, A^+\right] = 1$$

is treated as an operator of the creation of a universe with positive energy; and the coefficient A^-, as an operator of annihilation of an anti-universe also with positive energy. The physical states are formed by the action of these operators on vacuum $< 0|, |0 >$ in the form of out-state ($|1 >= A^+|0 >$) with positive frequencies and in-state ($< 1| =< 0|A^-$) with negative frequencies. This treatment means that positive frequencies propagate forward ($X_0 > X_{0I}$); and negative frequencies, backward ($X_0 > X_{0I}$) so that the negative values of energy are excluded from the spectrum to provide the stability of a quantum system [56].

The causal Green function is defined as the probability to find a universe at the moment X_0:

$$\begin{aligned} G^c(X_0) = \quad & < 0|T(\Psi(X_0)\Psi(0))|0 > \\ & \equiv G_+(X_0)\theta(X_0) + G_-(X_0)\theta(-X_0) \\ & = \frac{i}{2\pi}\frac{\exp(-iP_0X_0)}{P_0^2 - 4\gamma^2H_0^2 - i\epsilon}, \end{aligned} \qquad (107)$$

where $G_+(X_0) = G_-(-X_0)$ is the "commutative" Green function [56]

$$G_+(X_0) = \exp(-iP_0X_0)\delta(P^2 - 4\gamma^2H_0^2)\theta(P_0) \ . \qquad (108)$$

We see that the field variable X_0 plays the role of the evolution parameter that is the analogue of the time in the rest frame in Special Relativity. In both the cases Cosmic Relativity and Special Relativity the invariance of actions with respect to reparametrizations of the coordinate time means that one of dynamic variables becomes a parameter of evolution. Recall that this identification was the main feature of the approach of Poincare [23] and Einstein [24] to the solution of the problem of energy of a relativistic particle in Special Relativity. Their result is well-known in the form of $E = mc^2$ as the basis of nuclear power engineering.

The wave function (105) in the gauge $e_0 = 1$ does not contain the most interesting physical information about the evolution of the universe discussed in Section 2.9. This means that the quantum description of the universe in gauge $e_0 = 1$ is not complete. How can the evolution of the quantum universe with respect to the time measured by an observer be described?

C. Incorporation of time into quantum universe

To give the total description of the quantum universe, we create the time by the Levi-Civita canonical transformation [28, 31, 32, 34] of the *world field space* into *world geometric space*

$$(P_0, X_0) \Rightarrow (\Pi_0, Q_0) \qquad (109)$$

to new variables (Π_0, Q_0) for which one of equations identifies new scale factor Q_0 with the geometric interval Ω.

This transformation [31] is chosen so that to convert the constraint into the new momentum

$$\Pi_0 = \frac{P_0^2}{4\gamma} \ , \qquad Q_0 = X_0\frac{2\gamma}{P_0} \ . \qquad (110)$$

After transformation (110), the action (95) takes the form

$$S = \int\limits_{x_1^0}^{x_2^0} dx^0 \left[-\Pi_0\frac{dQ_0}{dx^0} - e_0(-\Pi_0 + \gamma H_0^2) - \frac{d}{dx^0}s^{lc}\right] \ , \qquad (111)$$

where $s^{lc} = (Q_0\Pi_0)$ is the generating functional of the canonical transformation.

We can check that the equation of motion for the momentum Π_0

$$\frac{\delta S}{\delta \Pi_0} = 0 \ \Rightarrow \ dQ_0 = e_0dx^0 \equiv d\Omega. \qquad (112)$$

identifies the dynamic evolution parameter Q_0 with the geometric interval Ω.

The solution of the equation of e_0 (i.e., the constraint)

$$-\Pi_0 + \gamma H_0^2 = 0 \ \Rightarrow \ \Pi_0 = \gamma H_0^2 \qquad (113)$$

determines a new Hamiltonian of evolution with respect to the new dynamic evolution parameter Ω. This Hamiltonian is not equal to zero in contrast to the gauge-fixing way of the description of a universe.

The substitution of all geometric solutions (113), (112)

$$Q_0 = \Omega, \ \ \Pi_0 = \gamma H_0^2 \qquad (114)$$

into the inverted Levi-Civita transformation (115)

$$P_0 = \pm 2\sqrt{\Pi_0\gamma} \ , \qquad X_0 = \pm Q_0\sqrt{\frac{\Pi_0}{\gamma}} \qquad (115)$$

leads to the conventional relativistic solution for the field system (100)

$$P_0 = \pm 2\gamma H_0 \ , \qquad (116)$$

$$X_0(\Omega) = X_0(0) + \Omega\frac{P_0}{2\gamma} \ .$$

The quantization $-i[\Pi, Q_0] = 1$ means that instead of constraint (113) we have the Schrödinger equation for the wave function

$$\frac{d}{id\Omega}\Psi_{lc}(\Omega) = \gamma H_0^2 \Psi_{lc}(\Omega), \tag{117}$$

$$\Psi_{lc}(\Omega) = \exp(i\Omega\gamma H_0^2)\exp(is^{lc}) = \exp(+i2\Omega\gamma H_0^2)$$

that contains only one eigenvalue γH_0^2. We see that there are differences between the *field* (Poincare-Einstein) and *geometric* (Levi-Civita) descriptions. The field evolution parameter is given in the whole region $-\infty < X_0 < +\infty$, whereas the geometric one is only positive $0 < \Omega < +\infty$, as it follows from the properties of the causal Green function (107) after the Levi-Civita transformation (110)

$$G^c(Q_0) = \frac{i}{2\pi}\int\limits_{-\infty}^{+\infty} d\Pi_0 \frac{\exp(iQ_0\Pi_0)}{(\Pi_0 - \gamma H_0^2 - i\epsilon)} =$$

$$= \theta(\Omega)\exp(iQ_0\gamma H_0^2), \qquad Q_0 = \Omega \ .$$

Two solutions of the constraint (a universe and an antiuniverse) in the *field space* correspond to a single solution in the *geometric space*.

For the causal convention (104), the *geometric time* Ω in classical solutions (100)

$$\Omega(X_0, X_{0I}) = \pm\frac{1}{H_0}(X_0 - X_{0I}) \geq 0 \tag{118}$$

is always positive as a consequence of the stability of the corresponding quantum system. For an Einstein observer, the negative time does not exist.

Thus, the reparametrization-invariant content of the equations of motion of a relativistic universe in terms of the geometric interval is covered by two systems: the field and geometric ones. The field system describes the secondary quantization and the derivation of the causal Green function that determines the arrow of the geometric interval (Ω). At the same time, the geometric set of variables includes the geometric interval ($Q_0 = \Omega$) into the number of measurable quantities, and it gives us a second wave function of a relativistic universe observed in the *world geometric space*.

The relations $X_0(Q_0)$ between these two wave functions in the form of the Levi-Civita canonical transformation are treated as pure relativistic effects. These relativistic effects could not be described by a single Newton-like system. Any gauge of the type $e_0 = 1$ is an attempt to reduce the relativistic system to a single Newton-like system. This gauge violates the reparametrization invariance and loses part of physical information, in particular, the positive arrow of the geometric interval, the initial geometric data, and the fact of the origin of the time measured by the watch of a observer. They are just the cardinal problems of the modern cosmology: the positive arrow of the time, its origin, and the cosmic initial data.

D. Solution of the problem of cosmic singularity in quantum universe

To discuss the problem of cosmic singularity, we reconsider the problem of evolution of the inertial universe in the relative space-time

$$d\bar{s}^2 = (\bar{N}_0(x^0)dx^0)^2 - \sum_{i=1}^{3}(dx^i)^2 \ ,$$
$$\bar{N}_0(x^0)dx^0 \equiv d\eta \tag{119}$$

in terms of the running Planck mass

$$\varphi(x^0) = \varphi_0 a(x^0) = \varphi_0 e^{X_0(x^0)} \ . \tag{120}$$

In this case, the action (95) takes the form

$$S^F = \int\limits_{x_1^0}^{x_2^0} dx^0 \left[-P_\varphi\dot{\varphi} + \bar{N}_0\left(\frac{P_\varphi^2}{4V_0} - \rho_I(\varphi)V_0\right)\right] \ , \tag{121}$$

where $\rho_I(\varphi)$ is given by the "inertial" density that plays the role of the dark energy and almost coincides with the critical density at the present-day time

$$\rho_I(\varphi)|_{\varphi=\varphi_0} = \frac{H_0^2\varphi_0^4}{\varphi^2(\eta)}|_{\eta=\eta_0} = H_0^2\varphi_0^2$$
$$\equiv H_0^2 M_{\text{Planck}}^2\frac{3}{8\pi} \equiv \rho_{cr} \ . \tag{122}$$

The equation of motion in the stiff state

$$\frac{d\varphi}{N_0 dx^0} \equiv \varphi' = \frac{P_\varphi}{2V_0} = \pm\frac{H_0\varphi_0^2}{\varphi} \ \Rightarrow \ (\varphi^2)'' = 0$$

reproduces the square root dynamics (81)

$$\varphi(\eta) = \varphi_I\sqrt{1 + 2H_I\eta} \tag{123}$$

that describes SN data, as we have seen before in Section 2.9. The universe at the beginning of the evolution $\eta = 0$ is characterized by two vacuum data: a primordial value of the dynamic Planck mass and the primordial Hubble parameter

$$\varphi(\eta = 0) = \varphi_I, \qquad H(\eta = 0) = H_I \ . \tag{124}$$

These primordial data are connected with the present-day values of the dynamic Planck mass φ_0 and the Hubble parameter H_0 by the integral of motion

$$\varphi^2(\eta)H(\eta) = \varphi_I^2 H_I = \varphi_0^2 H_0 = \text{constant} \ . \tag{125}$$

The quantization of the dynamic Planck mass $i[\varphi, P_\varphi] = \hbar$ leads to a quantum version of the energy constraint well-known as the Wheeler-DeWitt equation

$$\left[-\frac{P_\varphi^2}{4V_0} + \rho_I(\varphi)V_0\right]\Psi_{wdw} = 0 \ .$$

Two possible values of the momentum $P_\varphi = \pm 2V_0\sqrt{\rho_I(\varphi)} = \pm 2\gamma H_0/\varphi$ (where $\gamma = V_0\varphi_0^2$) correspond to two Wheeler-DeWitt wave functions

$$\begin{aligned}\Psi_{wdw}(\varphi_I,\varphi_0) &= A^+ e^{iS_+(\varphi_I,\varphi_0)}\theta(\varphi_0-\varphi_I)\\ &+A^- e^{iS_-(\varphi_I,\varphi_0)}\theta(\varphi_I-\varphi_0)\ ,\end{aligned} \quad (126)$$

where $S_\pm(\varphi_I,\varphi_0)$ coincides with the constrained action (121)

$$S^{(F)}(\mathrm{constraint}) = \mathrm{S}_\pm(\varphi_\mathrm{I},\varphi_0) = \mp 2\gamma \mathrm{H}_0 \log\frac{\varphi_0}{\varphi_\mathrm{I}} \quad (127)$$

that keeps only the field variable. By analogy with a relativistic particle we can treat the coefficients A^+, A^- of the decomposition as the operator of creation of the universe and the operator of annihilation. The singularity $\varphi_0 \to 0$ is contained in the second solution.

Thus, the problem of singularity is solved in quantum theory by the nonzero initial data (124) for the solution that corresponds to creation of the universe.

The WDW wave function (126) is not complete as it loses the dependence of masses on the time (123). The complete description of the quantum relativistic universe is possible by two wave functions in two sets of variables: the *field system* and the *geometric* one.

E. Description of SN data in the Quantum Universe

To find the second geometric system, we use the Levi-Civita canonical transformation [28, 30]

$$[P_\varphi,\ \ \varphi] \ \ \Rightarrow\ \ [\Pi_0,\ \ Q_0] \quad (128)$$

$$\varphi = \varphi_I \exp\left\{\pm\frac{Q_0}{\varphi_I}\sqrt{\frac{\Pi_0}{V_0}}\right\}, \quad (129)$$

$$P_\varphi = \pm\sqrt{\Pi_0 V_0}\exp\left\{\mp\frac{Q_0}{\varphi_I}\sqrt{\frac{\Pi_0}{V_0}}\right\},$$

$$\bar{N}_0 = e_0 \exp\left\{\pm\frac{2Q_0}{\varphi_I}\sqrt{\frac{\Pi_0}{V_0}}\right\}$$

that incorporates geometric interval into the set of the geometric variables. We called this set the geometric system. In terms of the new geometric variables the action of the relativistic inertial universe takes the form (111).

Finally, we get the set of the geometric variables for which the energy constraint coincides with the momentum $\Pi_0 = \gamma H_0^2$, and the equation for this momentum Π_0 points out that the new variable coincides with the definition of the geometric interval

$$dQ_0 = e_0 dx^0 = \frac{d\eta}{1+2H_I\eta} \quad (130)$$

leading to

$$Q_0(\eta) = Q_I + \frac{1}{2H_I}\log(1+2H_I\eta)\ .$$

The corresponding wave function satisfies equation $\Pi_0\Psi_G(Q_0) = \gamma H_0^2\Psi_G(Q_0)$. The solution of this equation takes the form

$$\Psi_G[Q_0(\eta)] = \exp\left\{-i2Q_0(\eta)\gamma H_0^2\right\}\ . \quad (131)$$

The evolution of the universe (123) is defined as a relation between the spectral parameter φ of the WDW wave function (126) of the universe in the *world field space* and the spectral parameter $Q_0(\eta)$ of the wave function (131) of the same universe in the *world geometric space*. This relation (described by the Levi-Civita transformations (129)) gives us the dynamic status of the Hubble law in the quantum universe as a pure relativistic effect.

In the considered case of the inertial motion in the coset A(4)/L, this Hubble law in the quantum universe (123) is compatible with the SN data.

Due to the Levi-Civita relation the causal quantization of the field spectral parameter φ gives the region of definition of the geometric spectral parameter η as the positive arrow of the time

$$\eta_0 - \eta_I \geq 0 \quad (\varphi_0 \geq \varphi_I) \quad P_\varphi \geq 0\ , \quad (132)$$

$$\eta_0 - \eta_I \geq 0 \quad (\varphi_I \geq \varphi_0) \quad P_\varphi \leq 0\ .$$

Thus, the positive arrow of the time and its beginning can be considered as the evidences for the quantum nature of the universe.

The relativistic gauge-invariant description also solves the problem of positive energy in quantum theory of gravitation as the negative sign in (127) corresponds to annihilation of the universe. If we lived in the created universe, we should choose the positive sign.

We can see that the problems of cosmic initial data and positive arrow of time cannot be solved by the standard description of quantum universe by the single WDW wave function.

IV. CREATION OF MATTER FROM VACUUM

A. Statement of the problem

Recall that in the inflational models [57] it is proposed that from the very beginning the universe is a hot fireball of massless particles that undergo a set of phase transitions. However, the origin of particles is an open question as the isotropic evolution of the universe cannot create massless particles. Nowadays, it is evident that the problem of the cosmological creation of matter from vacuum is beyond the scope of the inflational models.

Here we try to explain the cosmological creation of particles from vacuum in the regime of inertial motion of the universe along geodesic in the coset A(4)/L in the framework of the conformal cosmology. There, the cosmic evolution in the Standard Model (SM) $S_{\rm SM}[F|M_{\rm Higgs}]$ is separated by the conformal transformation $S_{\rm SM}[\bar{F}|a(\eta)M_{\rm Higgs}]$. In this case, the spontaneous SU(2) symmetry breaking and the vacuum expectation value of the *relative* Higgs field $<\bar{\Phi}>=a(\eta)<\Phi>$ is determined by the cosmic dynamics of the scale factor with the nonzero initial data described before in the regime of the inertial motion in the coset $A(4)/L$.

The SN data and the chemical evolution of matter show evidence of this regime in the origin of the universe. In this case, using the SM perturbation theory we should explain not only the cosmological creation of particles of observational matter from vacuum, but a primordial origin of the temperature of these particles.

There are arguments in the favour of that conformal cosmology can explain a cosmological creation of matter from vacuum in the regime of isotropic evolution of the universe as creation of vector bosons due to their mass singularity [37, 38]. The first estimations of this effect were made in [17, 39] in the conformal invariant models. Here, we present the theoretical foundation of the creation vector bosons in the Standard Model in the regime of the inertial motion in the coset $A(4)/L$.

B. Standard Model in Riemannian space

The matter field is included into the total action (71)

$$S_{\rm tot}[\{F\}|\varphi_0,c_0]=S_{\rm GR}[g|\varphi_0]+S_{\rm SM}[g,\{f\}|c_0] \qquad (133)$$

in the form of the Standard model of electroweak and strong interactions with the set of fields $\{F\}=g,\{f\}$ and the Higgs parameter c_0. The corresponding action takes the form

$$S_{\rm tot}=\int d^4x\sqrt{-g}\left[L_{\Phi,\varphi_0}+L_g+L_l+L_{l\Phi}+...\right], \qquad (134)$$

where

$$\begin{aligned} L_{\Phi,\varphi_0} = &\ \frac{|\Phi|^2-\varphi_0^2}{6}R \\ &+D^-_\mu\Phi(D^{\mu,-}\Phi)^*-\lambda\left(|\Phi|^2-c_0^2\right)^2 \end{aligned} \qquad (135)$$

is the Lagrangian of Higgs fields with

$$D^-_\mu\Phi=(\partial_\mu-\imath g\frac{\tau_a}{2}A_\mu-\frac{\imath}{2}g'B_\mu)\Phi$$

and

$$\Phi=\left(\begin{array}{c}\Phi_+\\ \Phi_0\end{array}\right),$$

$$|\Phi|^2=\Phi_+\Phi_-+\Phi_0\bar{\Phi}_0; \qquad (136)$$

$$\begin{aligned} L_g =&\ -\frac{1}{4}\left(\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g\varepsilon_{abc}A^b_\mu A^c_\nu\right)^2 \\ &-\frac{1}{4}\left(\partial_\mu B_\nu-\partial_\nu B_\mu\right)^2 \end{aligned}$$

is the Lagrangians of gauge fields,

$$\begin{aligned} L_l =&\ \imath\bar{L}\gamma^\mu D^+_\mu L+\imath\bar{e}_R\gamma^\mu\left(D^F_\mu+\imath g'B_\mu\right)e_R \\ &+\bar{\nu}_R\imath\gamma^\mu\partial_\mu\nu_R \end{aligned} \qquad (137)$$

and that of the leptons

$$L=\left(\begin{array}{c}\nu_{e,L}\\ e_L\end{array}\right);\quad e_R,\nu_{e,R}\ ; \qquad (138)$$

D^F is the Fock derivative,

$$D^+_\mu L=(D^F_\mu-\imath g\frac{\tau_a}{2}A_\mu+\frac{\imath}{2}g'B_\mu)L.$$

The Lagrangian describing mass terms of leptons including that of the neutrino (if any) is

$$\begin{aligned} L_{l\varphi} =&\ -y_e\left(e_R\Phi^+L+\bar{L}\Phi e_R\right) \\ &-y_\nu\left(\bar{\nu}_R\Phi^+_CL+\bar{L}\Phi_C\nu_R\right), \end{aligned} \qquad (139)$$

where

$$\Phi_C=\imath\tau_2\Phi^+_C=\left(\begin{array}{c}\Phi^*_0\\ -\Phi_-\end{array}\right) \qquad (140)$$

and y_f are dimensionless parameters. For simplicity, we omitted the strong interaction sector.

C. Higgs effect in the inertial universe

The evolution of the universe in terms of the mass-scale factor

$$\varphi(x^0)=\varphi_0a(x^0), \qquad \varphi_0=M_{\rm Planck}\sqrt{\frac{3}{8\pi}} \qquad (141)$$

is separated by the Lichnerowicz transformations (28)

$$\begin{aligned} S_{\rm tot}[\{F\}|\varphi_0,c_0] =&\ S_{\rm tot}[\{\bar{F}\}|\varphi,y_0\varphi] \\ &+S_{\rm universe}[\varphi,\bar{N}_0]\ , \end{aligned} \qquad (142)$$

where $\bar{F}$ are observable fields and

$$y_0=\frac{c_0}{\varphi_0} \qquad (143)$$

is the Higgs parameter in the Planck mass - units,

$$S_{\rm universe}[\varphi,\bar{N}_0]=-V_0\int\limits_{x^0_1}^{x^0_2}dx^0\left[\frac{(\partial_0\varphi)^2}{\bar{N}_0}+\bar{N}_0\rho_I(\varphi)\right] \qquad (144)$$

is the action of a collective inertial motion of the universe along the geodesic line of the field space with the primordial density

$$\rho_I(\varphi) \sim \frac{\varphi_0^2\rho_{\rm cr.}}{\varphi^2} \ , \tag{145}$$

is the primordial value of the mass-scale φ_I, and $\bar{N}_0$ is the global lapse function that determines the invariant conformal time

$$d\eta = \bar{N}_0 dx^0 \ . \tag{146}$$

It is just the time measured by the watch of an observer far from heavy masses where the Einstein relative interval takes a flat form

$$d\bar{s}^2 = d\eta^2 - dx_i^2 \ . \tag{147}$$

In this case, the Higgs potential

$$L_{\rm Higgs} = -\lambda\left(|\bar\Phi|^2 - y_0^2\varphi^2\right)^2$$

describes the Higgs effect of the spontaneous SU(2) symmetry breaking

$$\frac{\partial L_{\rm Higgs}}{\partial|\bar\Phi|} = 0 \ \Rightarrow |\bar\Phi|_1 = 0, \tag{148}$$

$$|\bar\Phi|_{2,3} = \pm y_0\varphi \sim 10^{-17}\varphi \ .$$

Two last stable solutions (148) form dynamic masses of elementary particle in the units of the dynamic Planck mass $\varphi^2 = \varphi_I^2(1+2H_0\eta)$, in the inertial universe (144).

The masses of elementary particles in the Lagrangian of SM

$$L = y_e|\bar\Phi|\bar{e}e + ...,$$

remind the evolution of the universe.

D. Perturbation theory

The first step of the perturbation theory is to consider the independent "free" fields in the linear approximation of their equations of motion.

The perturbation theory in the relative field space [27] begins from the metric

$$ds^2 = \frac{\varphi^2(x^0)}{\varphi_0^2} d\bar{s}^2 \ , \tag{149}$$

where the relative interval reads

$$d\bar{s}_0^2 = d\eta^2 - [\delta_{ij} + 2h_{ij}]dx^i dx^j \ , \tag{150}$$

where

$$d\eta = \bar{N}_0(x^0)dx^0 \ .$$

We keep only independent local field variables $h_{ii} = 0, \partial_i h_{ij} = 0$ which are determined by independent initial values. All nonphysical variables (for which the initial values depend on other data) are excluded by the local constraint.

The substitution of the ansatz (149), (150) into the action (134) leads to the action of free fields in terms of physical variables [27, 38]

$$S_{\rm tot} = \int\limits_{x_1^0}^{x_2^0} dx^0 \bar{N}_0 \left[-V_0\left(\frac{d\varphi}{\bar{N}_0 dx^0}\right)^2 + V_0\rho_I(\varphi) + L\right], \tag{151}$$

where V_0 is a finite spatial volume, and L is the sum $L = L_0 + L_{\rm int}$ of the Lagrangian of interaction $L_{\rm int}$ and the free one

$$L_0 = \int\limits_{V_0} d^3x \left(\mathcal{L}_\chi + \mathcal{L}^\perp_{\rm vec} + \mathcal{L}^{||}_{\rm vec} + \mathcal{L}_{\rm rad} + \mathcal{L}_{\rm s} + \mathcal{L}_{\rm h}\right) \tag{152}$$

is the total Lagrangian of free fields. In particular,

$$\mathcal{L}_\chi = \left\{\chi'^2 + \chi_i\left[\vec\partial^2 - 4\lambda(y_h\varphi)^2\right]\chi_i\right\} \tag{153}$$

is the Lagrangian of a deviation of the modulus of the Higgs field $|\bar\Phi| = y_0\varphi + \chi$,

and

$$\mathcal{L}^\perp_{\rm vec} = \frac{1}{2}\left\{v'^{\perp 2}_i + v^\perp_i\left[\vec\partial^2 - (y_v\varphi)^2\right]v^\perp_i\right\} \ ,$$

$$\mathcal{L}^{||}_{\rm vec} = -\frac{(y_v\varphi)^2}{2}\left[v'^{||}_i\frac{1}{\left(\vec\partial^2 - (y_v\varphi)^2\right)}v'^{||}_i + v^{||2}_i\right] \tag{154}$$

are Lagrangians of the transverse ($\mathcal{L}^\perp_{\rm vec}$) and longitudinal ($\mathcal{L}^{||}_{\rm vec}$) components of the W- and Z- bosons [37, 38]. The Lagrangian of the fermionic spinor fields is given by

$$\mathcal{L}_s = \bar\psi\left\{-y_s\varphi - i\gamma_0\partial_\eta + i\gamma_j\partial_j\right\}\psi \ , \tag{155}$$

where the role of the masses is played by the dynamic Planck mass φ multiplied by dimensionless constants $y_{v,s}$; $\mathcal{L}_{\rm rad}$ is the Lagrangian of massless fields (photons γ, neutrinos ν) with $y_\gamma = y_\nu = 0$, and

$$\mathcal{L}_{\rm h} = \frac{\varphi^2}{24}\left\{\left(h'_{ij}\right)^2 - (\partial_k h_{ij})^2\right\} \tag{156}$$

is the Lagrangian of gravitons as weak transverse excitations of spatial metric $h_{ii} = 0$ with a unit determinant of the three-dimensional metric $\partial_j h_{ji} = 0$ (everywhere $f' = df/[\bar{N}_0 dx^0]$).

To find the evolution of all fields with respect to the proper time η, we use the Hamiltonian form of the action (134) in the approximation (150)

$$S_{\rm tot} = \int\limits_{x_1^0}^{x_2^0} dx^0 \left\{\int\limits_{V_0} d^3x \sum_F P_F\partial_0 f - P_\varphi\partial_0\varphi\right.$$

$$- \bar{N}_0 V_0 \left[-\frac{P_\varphi^2}{4V_0^2} + \rho_I(\varphi) + \rho_m(\varphi, F, P_F) \right] \Bigg\} \,, \quad (157)$$

where the Hamiltonian density

$$V_0 \rho_m(\varphi, F, P_F) = V_0 \rho_{(2)}(\varphi, F, P_F) + H_{\rm int} \quad (158)$$

is a sum of the densities $\rho_{(2)}(\varphi, F, P_F)$ of free fields F and their interactions. In particular, the massive scalar Higgs field is described by the Hamiltonian [27]

$$H_{(2)} = V_0 \rho_{(2)} = \frac{1}{2} \sum_k \left[P_{\chi,k}^2 + \omega_\chi^2(\varphi, k) \chi_k^2 \right] \,, \quad (159)$$

where $\omega_\chi^2(\varphi, k) = \sqrt{k^2 + y_\chi^2 \varphi^2}$.

The variation of the action with respect to the homogeneous lapse-function $\bar{N}_0$ yields the energy constraint

$$\frac{\delta W_0}{\delta N_0} = 0 \ \Rightarrow \ \varphi'^2 = \rho_I(\varphi) + \rho_m(\varphi) \,. \quad (160)$$

Recall that we supposed that $\rho_I(\varphi) \gg \rho_m(\varphi)$.

E. Hamiltonian of Quantum Field Theory in *world field space*

The substitution of the solution of the energy constraint

$$\frac{P_\varphi^2}{4V_0} = V_0 \rho_I + H_m \ \Rightarrow \ P_\varphi = \pm 2V_0 \sqrt{\rho_I + \rho_m} \quad (161)$$

into the action (157) leads to the Poincare-Einstein type actions with the field evolution parameter φ

$$S_\pm^{(F)} = \int\limits_{\varphi_I}^{\varphi_0} d\varphi \left\{ \sum_F P_F \frac{dF}{d\varphi} \mp 2V_0 \sqrt{\rho_I + \rho_m} \right\} \quad (162)$$

that describes the evolution of "free" massive fields with respect to the dynamic Planck mass.

If the inertial density is greater than matter, $\rho_I(\varphi) \gg \rho_m(\varphi)$, the actions (162) can be decomposed in a nonrelativistic form

$$2V_0 \sqrt{\rho_I + H_m/V_0} = 2V_0 \sqrt{\rho_I} + \frac{H_m}{\sqrt{\rho_I}} + \dots \,. \quad (163)$$

The substitution of (163) into (162) leads to the sum $S_\pm^{(F)} = S_\pm^{(I)} + S_\pm^{(m)}$ of the action of the inertial motion (127) $S_\pm^{(I)}$ and the action of matter

$$S_\pm^{(m)} = \int\limits_{\varphi_I}^{\varphi_0} d\varphi \left\{ \sum_F P_F \frac{dF}{d\varphi} \mp \frac{H_m}{\sqrt{\rho_I}} \right\} \,. \quad (164)$$

The neglect of the "back-reaction" allows us to use the solution of the constraint $P_\varphi = 2V_0 \sqrt{\rho_I(\varphi)}$

$$d\varphi(\eta) = \sqrt{\rho_I} d\eta$$

In this case the action (164) takes a form of the sum of the action of the inertial motion and the one of the matter field in the form of a standard action in quantum field theory

$$S_\pm^{(m)} = \int\limits_{\eta_I}^{\eta_0} d\eta \left\{ \sum_F P_F \frac{dF}{d\eta} \mp H_m \right\} \,. \quad (165)$$

Thus, we derived the ordinary action of QFT with the measurable conformal time η without the concept of nonlocal energy [43].

F. Holomorphic representation and number of particles

The relativistic system is given in the world field space (φ, F_k), where F_k are oscillators treated as "particles". We define "particles" as holomorphic field variables

$$F(t, \vec{x}) = \quad (166)$$

$$\sum_k \frac{C_F(\varphi) e^{ik_i x_i}}{\sqrt{2V_0 \omega_F(\varphi, k)}} \left(a_\sigma^+(-k, t) \epsilon_\sigma(-k) + a_\sigma(k, t) \epsilon_\sigma(k) \right) \,,$$

where

$$C_h(\varphi) = \frac{\sqrt{12}}{\varphi}, \quad C_v^{||}(\varphi) = \frac{\omega_v}{y_v \varphi},$$

$$C_\gamma(\varphi) = C_s(\varphi) = C_v^{\perp}(\varphi) = C_\chi(\varphi) = 1 \,.$$

These variables are distinguished by that they diagonalize the energy density of "free" fields (159)

$$\hat{\rho}_{(2)}(\varphi) = \sum_\varsigma \omega_F(\varphi, k) \hat{N}_\varsigma \,, \quad (167)$$

where $\omega_F(\varphi, k) = (k^2 + y_F^2 \varphi^2)^{1/2}$ is the one-particle energy; $\hat{N}_\varsigma = \frac{1}{2}(a_\varsigma^+ a_\varsigma + a_\varsigma a_\varsigma^+)$ is the number of particles; ς includes momenta k_i, species $F = h, \gamma, v, s$, and spins σ. As everyone can see the longidutial vector paricles have the mass singularity $m_v = y_v \varphi$ [37, 38].

Diagonalization of the density (167) can be treated as a rigourous definition of a "particle" in quantum field theory that is consistent with observational cosmology. Really, in cosmology we consider the universe as a collection of "particles" with definite energies.

Just for observable "particles" the equations of motion are not diagonal. In particular, after transformation (166) the canonical differential form in the action

$$\int\limits_{V_0} d^3x \sum_F P_F \partial_0 F = \sum_{\substack{\varsigma\, = \\ (k,F,\sigma)}} \frac{\imath}{2} (a_\varsigma^+ \partial_0 a_\varsigma - a_\varsigma \partial_0 a_\varsigma^+)$$

$$-\sum_{\varsigma}\frac{\imath}{2}(a^+_{\varsigma}a^+_{\varsigma}-a_{\varsigma}a_{\varsigma})\partial_0\Delta_{\varsigma}(\varphi)$$

acquires nondiagonal terms as sources of cosmic creation of particles.

The set of nondiagonal terms in SM is

$$\Delta_h(\varphi)=\log(\varphi/\varphi_I),\quad \Delta^{\perp}_v(\varphi)=\Delta_{\chi}(\varphi)=\frac{1}{2}\log(\omega_v/\omega_I),$$

$$\Delta^{||}_v(\varphi)=\Delta_h(\varphi)-\Delta^{\perp}_v(\varphi),$$

where φ_I and ω_I are initial values.

G. Cosmological creation of vector bosons

The number of created particles is calculated by diagonalization of the equations of motion by the Bogoliubov transformation

$$b_{\varsigma}=\cosh(r_{\varsigma})e^{i\theta_{\varsigma}}a_{\varsigma}+\imath\sinh(r_{\varsigma})e^{-i\theta_{\varsigma}}a^+_{\varsigma}\ . \qquad (168)$$

These transformations play the role of the Levi-Civita canonical transformation to the action-angle variables that give integrals of motion for the matter fields

$$\frac{d}{d\eta}b^+b=0\ . \qquad (169)$$

These vacuum cosmic initial data correspond to the state *nothing* $b|0>=0$.

The equations for the Bogoliubov coefficients

$$\begin{aligned}[\omega_{\varsigma}-\theta'_{\varsigma}]\sinh(2r_{\varsigma}) &= \Delta'_{\varsigma}\cos(2\theta_{\varsigma})\cosh(2r_{\varsigma}),\\ r'_{\varsigma} &= -\Delta'_{\varsigma}\sin(2\theta_{\varsigma})\end{aligned}$$

determine the number of particles

$$N_{\varsigma}(\eta)={}_{sq}\langle 0|\hat{N}_{\varsigma}|0\rangle_{sq}-1/2=\sinh^2 r_{\varsigma}(\eta) \qquad (170)$$

created during the time η from squeezed vacuum: $b_{\varsigma}|0\rangle_{sq}=0$ and the evolution of the density

$$\rho(\varphi)=\varphi'^2=\sum_{\varsigma}\omega_{\varsigma}(\varphi)_{sq}\langle 0|\hat{N}_{\varsigma}|0\rangle_{sq}\ .$$

The numerical solutions of the Bogoliubov equations for the time dependence of the vector boson distribution functions $N^{||}_v(k,\eta)$ and $N^{\perp}_v(k,\eta)$ are given in Fig. 1 (left panels) for the momentum $k=1.25H_I$. We can see that the longitudinal function is noticeably greater than the transverse one. The momentum dependence of these functions at the beginning of the universe is given on the right panels of Fig. 1. The upper panel shows us the intensive cosmological creation of the longitudinal bosons in comparison with the transverse ones. This fact is in agreement with the mass singularity of the longitudinal vector bosons discussed in [37, 38]. One of the features of this intensive creation is a high momentum tail of the momentum distribution of longitudinal bosons which leads to a divergence of the density of created particles defined as [58]

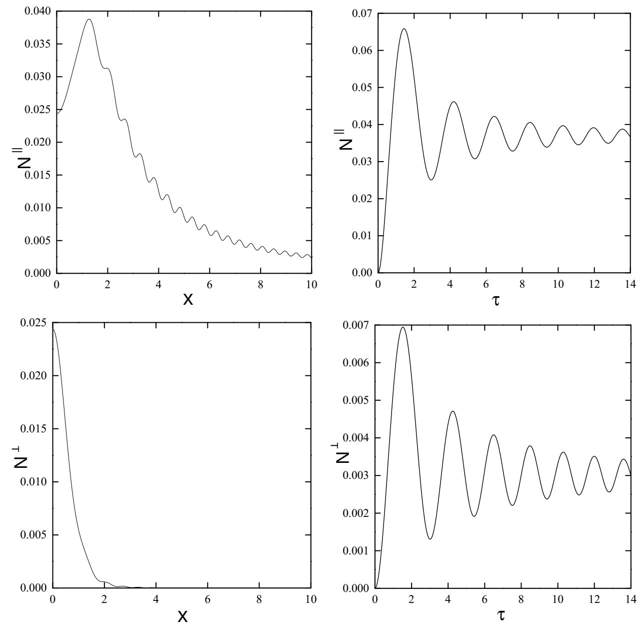

FIG. 1: Time dependence for the dimensionless momentum $x=k/H_I=1.25$ (left panels) and momentum dependence at the dimensionless lifetime $\tau=(\eta 2H_I)=14$ (right panels) of the transverse (lower panels) and longitudinal(upper panels) components of the vector-boson distribution function.

$$n_v(\eta)=\frac{1}{2\pi^2}\int\limits_0^{\infty}dkk^2\left[N^{||}_v(k,\eta)+2N^{\perp}_v(k,\eta)\right]\ . \qquad (171)$$

The divergence is a defect of our approximation where we neglected all interactions of vector bosons that form the collision integral in the kinetic equation for the distribution functions.

Our calculation of this density presented in Fig. 1 signals that the density (171) is established very quickly in comparison with the lifetime of bosons and in the equilibrium there is a weak dependence of the density on the time (or z-factor). This means that the initial Hubble parameter H_I almost coincides with the Hubble parameter at the point of saturation.

H. Towards the microscopic theory of CMB temperature

A new fact is the zero-mass singularity of the longitudinal vector bosons [37, 38] that leads to divergence of the number of created vector bosons in the lowest order

of perturbation theory [17]

$$n^{\|} = \frac{1}{2\pi^2} \int dk k^2 N^{\|}(k) \sim \infty \ . \qquad (172)$$

This divergence is the real origin of appearance of temperature of the created matter in the steady universe of the relative cosmology. This temperature belongs to vector bosons.

The concept of a *temperature* T for the matter fields in the relativistic region means that the distribution function $N = |\sinh^2 r|$ converts into the Boltzmann factor, and the particle density takes the form $n(T) \sim T^3$. This *temperature* is established due to the interaction cross-section

$$\sigma_{\rm scat.} \sim \frac{1}{m_v^2(\eta_{\rm relax.})}$$

during the time of relaxation

$$\eta_{\rm relax.} = \frac{1}{n(T)\sigma_{\rm scat.}} \ .$$

We can introduce the concept of *temperature* T if this time of relaxation is less than the inverse Hubble parameter (i.e., the universe is varying slowly than fields) $\eta_{\rm relax.} \leq 1/H(\eta_{\rm relax.})$. This means that the temperature T can be estimated by the integral of motion

$$n(T)\sigma_{\rm scat.} \geq H(\eta_{\rm relax.}) \ \Rightarrow$$

$$T_I \sim \left(m_v^2(\eta_{\rm relax.})H(\eta_{\rm relax.})\right)^{1/3} = \left(m_{v0}^2 H_0\right)^{1/3} \ , \quad (173)$$

where m_{v0}^2, H_0 mean the present-day values. If we suppose that the CMB radiation is the product of the decay of the primordial vector bosons and its temperature $T_{CMB} = 2.7K$ remembers the primordial temperature $T_I = T_{CMB} = 2.7K$, we can obtain the present-day mass of the primordial vector bosons

$$m_{v0} = \left(\frac{T_{CMB}^3}{H_0}\right)^{1/2} \simeq 80GeV \div 100GeV \ . \qquad (174)$$

It is just the mass of W, Z - vector bosons in the Standard Model of the electroweak interactions. Using this mass we can estimate also primordial values of the of Hubble parameter and Planck mass. They are close to

$$H_I = T_{CMB} = 2.7K, \ \ \varphi_I = \frac{\varphi_0 T_{CMB}^{1/2}}{H_0^{1/2}} = 10TeV \ . \ (175)$$

One can say that the CMB-temperature "remembers" the primordial mass of the inertial motion of the universe.

These estimations are in agreement with the latest Supernova data [16, 20, 21, 22], the chemical evolution, the positive arrow of the geometric time, and the inertial density

$$\rho_I(\varphi) = \frac{\varphi_I^4 T_{CMB}^2}{\varphi^2} \equiv \frac{\varphi_0^2 \rho_{\rm cr.}}{\varphi^2} \ .$$

It is varying in the time η from $10^{29}\rho_{\rm cr.}$ to $\rho_{\rm cr.}$ during the time $\eta = 1/2H_0$. This density is greater than the one of created bosons

$$\frac{\rho_I(\varphi_I)}{T_{\rm CMB}^4} = \frac{\varphi_I^2}{T_{\rm CMB}^2} = 10^{-34} \ .$$

These estimations are compatible with a direct calculation of the primordial creation of vector particles [17, 39]. This creation leads to the baryon asymmetry of the universe as the left-current interaction of the primordial vector bosons in SM during their lifetime polarize the Dirac sea of fermions [16].

V. DISCUSSION OF THE RESULTS

The universe was considered as one of ordinary physical objects described by differential equations of the General Relativity and Standard Model given in the definite frame of reference (connected with the CMB radiation) with the zero initial data for all fields $< \bar{F} >_I = 0$, excluding the scale factor

$$a_I = 0.3 \cdot 10^{-14} \ , \qquad (a'/a)_I = 2.7K$$

and the relative Higgs field

$$< \bar{\Phi} >_I = a_I < \Phi >_0 \ ,$$

where $< \Phi >_0$ is the present-day value.

It is useful to remind also that modern quantum field theories cannot explain a measurement standard (identifying the theoretical quantities with the observational ones) and initial data including the background of infrared gravitation fields (like an undetected background of infrared photons which should be taken into account for the consistent theoretical description of the higher energy accelerator experiments).

Therefore, the modern status of the theory allows us to consider the homogeneous approximation, the absolute status of measurement standard of the Paris meter, the Planck initial data $a_I \simeq 10^{-60}$ of origin of the matter, and the Higgs field (i.e., material) origin of the cosmic evolution of the scale factor as subjects of scientific research rather than dogmas.

In the paper we listed the arguments in favour of that the latest observational data fit the relative measurement standard (with the expanding Paris meter) and the vector boson initial data $a_I \simeq 10^{-14}$ of origin of the matter, and the gravitational origin of the cosmic evolution of the scale factor treated as the collective inertial motion along a geodesic in the field space of metric components.

The considered theory explains the creation of the universe in the world field space $(a|\bar{F})$ with the field evolution parameter a and the energy

$$P_a = E_{universe} + H_{\rm QFT} d\eta/da + .. \ ,$$

where

$$E_{universe} = 2V_0 \varphi_0^2 (H_I a_I^2) \log(a_0/a_I)$$

like the modern quantum field theory explains particle creation in the Minkowski world space $(X_0|X_i)$ with the energy

$$P_0 = E_{mass} + (P_i^2/2m)(d\eta/dX_0) + ..$$

(where $E_{mass} = mc^2$). In both the cases $d\eta$ means the geometric time interval.

The considered theory introduces the geometric time interval in the Hamiltonian description by the Levi-Civita canonical transformation of the field variables to the geometric set of variables. In this case, the causal quantization of the scale factor removing the negative energy P_a explains both the absence of the cosmological singularity $a = 0$ (in the field wave function of created universe) and the positive arrow of the geometric time η (in the geometric wave function of created universe) as the consequences of stability of quantum theory. The relation of two evolution parameters of these wave functions in the form of the evolution of the universe $a(\eta)$ is a pure relativistic effect. Just this relation gives the dynamic status of the Hubble law in the quantum universe.

The theory explains the origin of observational matter. Remind that the collective motion with the momentum P_a allows us to define energy of the universe compatible with the one in quantum field theory $H_{\rm QFT}$, the observational energy density, and observational particles.

The theory points out the creation of W, Z - vector bosons from the geometric vacuum due to their mass singularity $\bar{m}_W(a \to 0) \to 0$. This mass singularity is the physical origin of the temperature of the matter. The inertial cosmic motion in the coset $A(4)/L$ leads to the definite temperature of the matter $T_I \simeq (m_W^2 H)^{1/3} K$ depending on the boson mass m_W and the Hubble parameter H. This temperature is an integral of the inertial motion and this integral coincides with the primordial value of the Hubble parameter $T_I = H_I$.

These primordial bosons decay with the baryon number violation [39]. The CMB radiation as the product of decay of the primordial bosons keeps the primordial value of the temperature $T_{CMB} \simeq (m_{W0}^2 H_0)^{1/3} = 2.7K$, where m_{W0}, H_0 are the present-day values of boson mass and the Hubble parameter in the stiff regime.

Thus, we have shown that the quantum universe could be created in the CMBR reference frame with the value of the primordial scale factor $a_I \approx 0.3 \times 10^{-14}$ and the primordial Hubble parameter $H_I \approx 2.7K$.

VI. CONCLUSION

The treatment of General Relativity as the theory of nonlinear representation of the affine group helped us to determine the geometry of the field space, and extended the principles of relativity (including the concepts of the inertial motion, geodesic line, relative and absolute "coordinates") to this field space.

The consistent and complete description of the creation of the quantum universe and its evolution allowed us to consider the creation of matter in the quantum universe in the Standard Model of the electroweak and strong interactions.

This theoretical description depends on the Lorentz frame. In this sense the explicit relativistic covariance is lost. Remind that this fact is not a defect of the theory. The relativistic covariance means that a complete set of states obtained by all Lorentz transformations of a state in a definite frame of reference coincides with a complete set of states obtained by all Lorentz transformations of this state in another frame of reference (see the review of papers by V. Bargmann, E.P. Wigner, and A.S. Wightman in the monography [56]).

Moreover, following to Julian Schwinger "...we reject all Lorentz gauge formulations as unsuited to the role of providing the fundamental operator quantization ..." [59] counting that any gauges that do not depend on a reference frame are not correct for the description of amplitudes of all processes in quantum theory, including the creation of the universe, time, and matter with the exception of the narrow class of amplitudes of the scattering of elementary local fields on their mass-shell [54, 60].

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Appendix A: Hamiltonian formalism in the coset A(4)/L

A choice of a Lorentz-frame in GR means the separation of all underlined indices into the time-like and space-like ones ($\underline{\mu} = \underline{0}, \underline{a}$).

To formulate Hamiltonian dynamics in GR, besides of the Lorentz-frame in the Minkowski space-time $\underline{\mu}$ we should choose also the **Riemannian frame**. The Riemannian frame of reference for solving the evolution problem in GR is known as the "kinemetric" frame [8]. This frame means the $3 + 1$ foliation of the Riemannian space-time

$$\omega_{\underline{0}} = N dx^0, \qquad \omega_{\underline{a}} = e_{\underline{a}i}(dx^i + N^i dx^0) \ ,$$

where $e_{\underline{a}i}$ is "drei-bein". This foliation in terms of an Einstein interval

$$(ds)^2 \ = \ g_{\mu\nu}dx^\mu dx^\nu = (N dx^0)^2 \\ -{}^{(3)}g_{ij}\left(dx^i + N^i dx^0\right)\left(dx^j + N^j dx^0\right) \quad (A.1)$$

was applied for the generalized Hamiltonian approach to the Einstein theory of gravitation (16) by Dirac and

Arnowitt, Deser and Misner [41, 42]. This foliation keeps all ten components of the metric with the lapse function $N(x^0,\vec{x})$, three shift vectors $N^i(x^0,\vec{x})$, and six space components ${}^{(3)}g_{ij}(x^0,\vec{x})$ depending on the coordinate time x^0 and space coordinates $\vec{x}$. The Dirac-ADM parametrization characterizes a family of hypersurfaces $x^0 = \text{const.}$ with the unit normal vector $\nu^\alpha = (1/N, -N^k/N)$ to a hypersurface. The second (external) form

$$\frac{1}{N}\left(\partial_0{}^{(3)}g_{ij} - N_{j|i} - N_{i|j}\right) \equiv$$

$$\frac{1}{N}\left[\left(\partial_0 - N^l\partial_l\right){}^{(3)}g_{ij} - {}^{(3)}g_{il}\partial^k N^l - {}^{(3)}g_{jl}\partial_i N^l\right] \quad (A.2)$$

shows how this hypersurface is embedded into the four-dimensional space-time. Here $N_{i|j}$ is the covariant derivative with respect to the metric ${}^{(3)}g^{kj}$. In terms of "drei-bein" the expression (A.2) takes the form

$$\left(\partial_0{}^{(3)}g_{ij} - N_{j|i} - N_{i|j}\right) \equiv e_{\underline{a}i}(D_0 e)_{\underline{a}j} + (i \leftrightarrow j) \ , \quad (A.3)$$

where

$$(D_0 e)_{\underline{a}i} = \left(\partial_0 - N^l\partial_l\right) e_{\underline{a}i} - e_{\underline{a}l}\partial_i N^l \ . \quad (A.4)$$

A gauge group is considered as the group of diffeomorphisms of the Dirac-ADM parametrization of the metric (A.1) [8]

$$x^0 \to \tilde{x}^0 = \tilde{x}^0(x^0); \qquad x_i \to \tilde{x}_i = \tilde{x}_i(x^0, x_1, x_2, x_3) \ , \quad (A.5)$$

$$\tilde{N} = N\frac{dx^0}{d\tilde{x}^0}; \qquad \tilde{N}^k = N^i\frac{\partial\tilde{x}^k}{\partial x_i}\frac{dx^0}{d\tilde{x}^0} - \frac{\partial\tilde{x}^k}{\partial x_i}\frac{\partial x^i}{\partial\tilde{x}^0} \ . \quad (A.6)$$

These transformations conserve the family of hypersurfaces $x^0 = \text{const.}$, and they are called a kinemetric subgroup [8, 26, 27] of the group of general coordinate transformations (19). The group of kinemetric transformations contains reparametrizations of the *coordinate time* (A.5). This means that there are no physical instruments that can measure this *coordinate time*. The definition of the kinemetric frame of reference requires to point out an **invariant field evolution parameter** in the field space $g_{\mu\nu} = (g_{ij}, N, N^k)$ which plays the role of the fourth time-like coordinate in Minkowskian space-time in Special Relativity.

The Einstein action in the kinemetric frame takes the form

$$S_{\rm GR}[e|\varphi_0] = -\int d^4x\sqrt{-\bar{g}}\frac{\varphi_0^2}{6}R(g) =$$

$$\int\limits_{x_1^0}^{x_2^0} dx^0 \int\limits_{V_0} d^3x \left[\mathtt{K}(e|\varphi_0) - \mathtt{P}(e|\varphi_0) + \mathtt{S}(e|\varphi_0)\right] \ , \quad (A.7)$$

where

$$\mathtt{K}(e|\varphi_0) = \frac{\varphi_0^2|e|}{24N}[\pi_{\underline{ab}}\pi_{\underline{ab}} - \pi_{\underline{bb}}\pi_{\underline{aa}}] \quad (A.8)$$

is the kinetic term,

$$\mathtt{P}(e|\varphi_0) = \frac{\varphi_0^2 N|e|}{6}\ {}^{(3)}R(e) \quad (A.9)$$

is the potential term with a three dimensional curvature in terms of "drei-beins" $e_{\underline{a}i}$, and

$$\mathtt{S}(e|\varphi_0) = \frac{\varphi_0^2}{6}\left(\partial_0 - \partial_k N^k\right)\left(\frac{|e|\pi_{\underline{aa}}}{N}\right) - \frac{\varphi_0^2}{3}\partial_i\left(|e|g^{ij}\partial_j N\right) \quad (A.10)$$

are the standard ADM "surface terms" which do not contribute to the equations of motion. Here we used the following definitions

$$\sqrt{{}^{(3)}\bar{g}} = det||e_{\underline{a}i}|| \ , \quad g^{ij} = (e^{-1})_{i\underline{a}}\ (e^{-1})_{j\underline{a}} \ , \quad (A.11)$$

$$\pi_{\underline{ab}} = \omega^R_{\underline{ab}}(D_0) = \frac{1}{2}[(D_0 e)_{\underline{a}i}(e^{-1})_{i\underline{b}} + (\underline{a} \leftrightarrow \underline{b})]) \ .$$

The definition of $\omega^R_{(\underline{ab})}(D_0)$ is given by (7) where (de) is replaced by the covariant derivative (A.4).

One can choose a triangle "drei-bein"

$$e_{\underline{a}i} = 0, \qquad \underline{a} < i \quad (A.12)$$

that is the continuation of the $4 = 3 + 1$ foliation for the $3 = 1 + 1 + 1$ one.

To make the discussion of the **invariant field evolution parameter** in GR more transparent, it is useful to separate the determinant of the three-dimensional metric [47]

$$e_{\underline{a}i} = \psi^2 e^T_{\underline{a}i} \ , \quad det||e^T_{\underline{a}i}|| = 1 \ , \quad det||g^T_{ij}|| = 1. \quad (A.13)$$

Then, instead of the Lagrangians (A.8), (A.9) and (A.10) we get the kinetic term

$$\mathtt{K}(g|\varphi_0) = \frac{\varphi_0^2\psi^6}{N}\left[\frac{\pi^T_{\underline{ab}}\pi^T_{\underline{ab}}}{24} - 4\pi_\psi^2\right] \ , \quad (A.14)$$

where

$$\pi_\psi = (\partial_0 - N^k\partial_k)\log\psi - \frac{1}{6}\partial_k N^k \ , \quad (A.15)$$

the potential term is

$$\mathtt{P}(g|\varphi_0) = \frac{\varphi_0^2 N\psi^2}{6}\left[{}^{(3)}R(g_{(T)}) + \frac{8}{\psi}g^{ij}_{(T)}\partial_i\partial_j\psi\right], \quad (A.16)$$

and the "surface" term is

$$\mathtt{S}(g|\varphi_0) = \varphi_0^2 2\left(\partial_0 - \partial_k N^k\right)\left(\frac{\psi^6\pi_\psi}{N}\right) \quad (A.17)$$

$$-\frac{\varphi_0^2}{3} g^{ij}_{(T)} \partial_i \left(\psi^2 \partial_j N\right) \ .$$

Now we can introduce canonical momentum using the standard definitions and eqs. (A.11), (A.14)

$$P_\psi = \frac{\partial \mathrm{K}(g|\varphi_0)}{\partial(\partial_0 \psi)} = -8\varphi_0^2 \psi^5 \pi_\psi \ , \tag{A.18}$$

$$P_{\underline{ab}} = \frac{1}{2}\left[e^T_{\underline{a}i} P^i_{\underline{b}} + (\underline{a} \leftrightarrow \underline{b})\right] \ , \tag{A.19}$$

$$P^i_{\underline{a}} = \frac{\partial \mathrm{K}(g|\varphi_0)}{\partial(\partial_0 e_{\underline{a}i})} = \frac{\varphi_0^2 \psi^6}{12N}\left[(e_T^{-1})_{\underline{a}i} \pi^T_{\underline{ab}}\right] \ .$$

In terms of these momenta the Einstein action takes the first-order Hamiltonian form

$$S_{\mathrm{GR}}[F|\varphi_0] = S_F + S_{\mathrm{GR}} = \tag{A.20}$$

$$\int dx^0 d^3x \left[\sum_{F=\psi, e^T} P_F \partial_0 F - N\mathcal{H} + N^k \mathcal{P}_k + \mathtt{S}(g|\varphi_0)\right],$$

where the local Hamiltonian density

$$N\mathcal{H} = \mathtt{K}(g|\varphi_0) + \mathtt{P}(g|\varphi_0) \tag{A.21}$$

is the sum of terms defined by eqs. (A.14), (A.16), the kinetic Lagrangian

$$\mathtt{K}(g|\varphi_0) = N\left\{\frac{6}{\varphi_0^2 \psi^6}[P_{(\underline{ab})} P_{(\underline{ab})}] - \frac{1}{16\varphi_0^2 \psi^4} P_\psi^2\right\} \tag{A.22}$$

depends on the canonical momenta $P_{(\underline{ab})}$ and P_ψ, and

$$\mathcal{P}_k = \partial_j(P^i_{\underline{a}} e^T_{\underline{a}k}) + P^i_{\underline{a}} \partial_k e^T_{\underline{a}i} - T^0_k \tag{A.23}$$

is the local momentum where the energy momentum tensor that depends on the trace of the second external form

$$T^0_k = P_\psi \partial_k \psi - \frac{1}{6}\partial_k(P_\psi \psi) \tag{A.24}$$

is distinguished.

We have the Dirac generalized Hamiltonian system with four local constraints (A.21), (A.23)

$$\mathcal{H}_{\mathrm{tot}} = 0, \qquad \mathcal{P}_k = 0 \ . \tag{A.25}$$

In terms of “observables”

$$\mathcal{P}_{\underline{a}} = \mathcal{P}_k (e_T^{-1})_{k\underline{a}}$$

the second three constraints $\mathcal{P}_{\underline{a}} = 0$ take the transparent form of the transverseness condition

$$\partial_j P^j_{\underline{b}} = \tilde{T}^0_{\underline{b}} \ , \tag{A.26}$$

where

$$\tilde{T}^0_{\underline{b}} = T^0_k (e_T^{-1})_{k\underline{b}} + P^j_{\underline{a}}(\partial_k e^T_{\underline{a}j} - \partial_j e^T_{\underline{a}k})(e_T^{-1})_{k\underline{b}} \ .$$

Four constraints (A.25) should be accompanied by four gauges. Dirac [41] chose the so-called minimal embedding of a three-dimensional hypersurface into the four-dimensional space-time

$$P_\psi = 0 \tag{A.27}$$

and the transverseness condition that in terms of ”drei-beins” takes the form

$$f_{\underline{b}}(e^T) = 0 \ \Rightarrow\ \partial^i e^T_{\underline{b}i} = 0 \tag{A.28}$$

compatible with the constraint (A.26). The minimal embedding allowed Dirac to remove all local excitations of metric with the negative norm and a negative contribution to energy.

Faddeev and Popov [43] used other gauges. Using general coordinate transformations they removed all components of the three-dimensional metric besides two. Using general coordinate transformations one can also remove all components of ”drei-beins” besides two. In particular, we can keep two nondiagonal components in the triangle basis (A.12) of $(3 = 1 + 1 + 1)$ foliation

$$e^T_{\underline{a}i} = \begin{vmatrix} 1 & 0 & 0 \\ e_{\underline{2}1} & 1 & 0 \\ 0 & e_{\underline{3}2} & 1 \end{vmatrix} \ . \tag{A.29}$$

The triangle basis (A.29) allows us to find the polynomial form of the three-dimensional metric

$$g^T_{ij} = \begin{vmatrix} 1 & e_{\underline{2}1} & 0 \\ e_{\underline{2}1} & 1 + e^2_{\underline{2}1} & e_{\underline{3}2} \\ 0 & e_{\underline{3}2} & 1 + e^2_{\underline{3}2} \end{vmatrix} \ , \tag{A.30}$$

$$g^{ij}_T = \begin{vmatrix} 1 + e^2_{\underline{2}1}(1 + e^2_{\underline{3}2}) & -e_{\underline{2}1}(1 + e^2_{\underline{3}2}) & 0 \\ -e_{\underline{2}1}(1 + e^2_{\underline{3}2}) & 1 + e^2_{\underline{3}2} & -e_{\underline{3}2} \\ e_{\underline{2}1} e_{\underline{3}2} & -e_{\underline{3}2} & 1 \end{vmatrix} \tag{A.31}$$

for which the GR action becomes polynomial. In these cases, with the affine version of General Relativity one hopes to prove its renormalizability.

Thus, the Hamiltonian action (A.20) with all constraints takes the form

$$S_{\mathrm{GR}}[F|\varphi_0] = \tag{A.32}$$

$$\int dx^0 \left\{\left[\int d^3x \sum_{F=\psi, e,} P_F \partial_0 F\right] - H_{\mathrm{tot}}[F|\varphi_0]\right\} \ ,$$

where

$$H_{\mathrm{tot}}[F|\varphi_0] = \tag{A.33}$$

$$\int d^3x\,[N\mathcal{H}-N^k\mathcal{P}_k+C_0P_\psi+C_{\underline{b}}f_{\underline{b}}(e^T)-\mathtt{S}]$$

is the total Hamiltonian with the Lagrangian multipliers $N, N^k, C_0, C_{\underline{b}}$.

In the context of our consideration of the problem of energy and time it is very important to emphasize the positive contribution of the transverse "drei-beins" to the Hamiltonian density $\mathcal{H}_{\rm tot}=0$ in the conventional perturbative theory. The latter begins with the gauge

$$N=1, \qquad \psi=1 \qquad (A.34)$$

that supposes that the measurable time is identified with the coordinate time x^0. We see below that the gauge $N=1$ removes part of pure relativistic results including definition of nonzero energy of the proper time, a positive arrow of the time, and cosmic initial data.

The perturbation theory with $\psi=1$ loses all cosmology. Recall that the evolution of the universe is identified with the determinant of the spatial metrics in the homogeneous approximation $\psi^2=a(x^0)$ that is well known as cosmic scale factor.

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